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**Optimizing tank arrangement against sloshing loads for  
floating production storage and offloading unit**

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Sloshing is an important factor in FPSO vessel's tank structure because the fluid surface varies constantly in a tank due to operation. When the fluid surface varies it may cause a situation, in which natural periods of the fluid and vessel meet. If that happens, sloshing is at it's worst. The purpose of this thesis is to develop a concept design phase calculation procedure for sloshing analysis. The calculation procedure helps to optimize sloshing loads and to recognize when the sloshing pressure is a governing load in the vessel's tank structure.

A solution to the research problem is an enumeration method coupled with motion analysis and closed-form analytical equations. The idea is to delimit all possible tank sizes and then optimizing a suitable tank size for the vessel. The basis for calculation of sloshing pressure is the approach from ABS classification society's rules and especially the terms which include acceleration and natural period. This improves the accuracy of results. The accurate acceleration and natural period terms are calculated using an AQWA LINE program which allows the calculation of exact acceleration and natural period values at each point on the vessel. The next step is to calculate sloshing pressure.

The results show, that the method is feasible for concept design. Sloshing loads can be minimized reasonably without making the tank size too small. The case study vessel's results show that sloshing loads are dominating loads when the tank length is large. In this case sloshing causes a large pressure peak on top of the tank structure. In other cases the governing load is hydrodynamic pressure. The case study vessel already has five tanks and the results show that it is an optimum solution for sloshing loads. The results are more accurate than the classification society rules results. The ABS equations acceleration and natural period terms are exact values. As a result, it can be said that the accuracy of these results are better because the acceleration and natural period values are vessel-specific in defined sea conditions.

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**Keywords** Sloshing, ABS, DNV, IACS, FPSO, tank structure, Classification society

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FPSO-aluksen tankkirakenteessa loiskunta on tärkeä tekijä, koska nestepinta vaihtelee tankissa jatkuvasti. Nestepinnan vaihtelusta voi seurata, että nesteen ja laivan ominaisperiodit kohtaavat. Tällöin loiskunta on pahimmillaan. Tässä työssä ongelmana on, että ei ole tietoa miten loiskekuormat tulisi laskea tarkasti FPSO aluksen tankissa konseptisuunnittelu vaiheessa ja miten suuria loiskekuormia tankissa esiintyy. Tämän työn tarkoituksena on kehittää konseptisuunnitteluun laskentamalli loiskepaineiden ja taajuuksien laskemiseksi aluksen tankissa. Tämän avulla pystytään optimoimaan loiskekuormat ja tunnistamaan milloin loiskinta on aluksen tankkirakennetta mitoittava kuorma.

Tutkimusongelman ratkaisuna on enumeraatio menetelmän hyödyntäminen yhdistetynä laivan liikelaskentaan sekä analyttisiin paine- ja ominaistajuusmenetelmiin. Ideana on ensin rajata mahdolliset tankki koot ja tämän jälkeen optimoida alukselle sopiva tankin koko. Loiskekuormien laskennassa lähestymistapana on ABS luokitustilalaitoksen analyttiset menetelmät. Tarkentamalla luokituslaitoksen menetelmässä esiintyviä kiihtyvyyden ja ominaistajuus-arvoja, voidaan luokituslaitoksen säännöillä laskettuja tuloksia parantaa. Tarkempien kiihtyvyyden ja ominaistajuus arvojen laskemiseen käytetään hyödyksi AQWA LINE ohjelmaa. Mallin avulla pystytään laskemaan tarkat kiihtyvyyden arvot aluksen jokaisessa pisteessä. Oletuksena on, että nesteen kiihtyvyys on sama kuin aluksen kiihtyvyys samassa pisteessä. Laskemalla loiskepaineet erikokoisille tankeille, pystytään optimoimaan milloin loiskinta on tankkia mitoittava kuorma ja milloin tankin staattinen tai hydrodynaaminen paine on mitoittava kuorma.

Tulokset osoittavat, että loiskekuormat pystytään minimoimaan järkevästi niin, ettei tankeista tule liian pieniä. Tutkimuskohteena olevan aluksen tulokset osoittavat, että tankin pituuden ollessa suuri, loiskinta aiheuttaa tankin ylärakenteisiin tankkirakennetta mitoittavan paineen. Muissa tapauksissa tankin hydrodynaaminen paine on tankin mitoittava paine. Tutkimuskohde aluksessa oli valmiiksi 5 tankkia ja tulosten perusteella tämä on optimaalinen vaihtoehto loiskekuormien osalta. Aluksen loiskintakuormia on mietitty aluksen suunnittelussa. Luokituslaitoksen kaavoja on tarkennettu, jonka seurauksena saadaan tarkempia tuloksia aluskohtaisesti.

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**Avainsanat** Loiskunta, ABS, DNV, IACS, luokituslaitos, FPSO, tankkirakenne

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## **PREFACE**

This thesis has been carried out for Deltamarin Ltd during the year 2012-2013

I would like to thank Deltamarin for this opportunity to write my thesis in great company. I would like to give special thanks to my supervisor professor Jani Romanoff and instructors Timo Mikkola and Oskar Enqvist for their help, comments and support.

Finally, I would like to thank my family, friends and girlfriend. Who have supported me throughout the challenging times of making this thesis.

Espoo, April 23, 2014

Harri Suistio

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# LIST OF SYMBOLS

## Notations

$a_i$	Effective resultant acceleration
$B$	Ship breadth
$b_s$	Effective sloshing breadth
$c_m$	Place coefficient
$C_{\phi s}$	Weighted coefficient
$C_{\theta s}$	Weighted coefficient
$d_m$	Filling level for ABS
$d_{TB}$	Vertical distance from baseline to the tank bottom
$f$	The natural sloshing frequency
$g$	Gravitation acceleration
$GM$	Height of the ship's metacenter
$H$	Tank height
$h$	Filling level
$h$	Liquid depth
$h$	Tank depth for ABS
$h_c$	Maximum average sloshing pressure heads for ABS
$h_e$	Equivalent liquid pressure head for ABS

$h_b$	Average sloshing pressure head breadth
$h_d$	Wave-induced internal pressure head
$h_l$	Average sloshing pressure head length
$h_t$	Sloshing pressure head for upper bulkhead
$h_{tb}$	Sloshing pressure head for upper bulkhead breadth
$h_{tl}$	Sloshing pressure head for upper bulkhead length
$k_c$	Correlation factor for combined load cases
$k_f$	Factor for DNV rules
$k_s$	Load factor for ABS
$k_u$	Load factor for ABS
$L$	Ship length
$l_s$	Effective length
$l$	Tank length
$n$	Number of significant sloshing frequency*
$p$	Pressure
$p_0$	Reference pressure
$p_i$	Hydrodynamic pressure for ABS
$p_{is}$	Sloshing pressure for ABS
$p_s$	Still water pressure



$T_n$	The natural sloshing periods
$T_x$	The natural period of the fluid motion in longitudinal
$T_y$	The natural period of the fluid motion in transverse
$y$	Vertical coordinate for ABS rule
$z$	Vertical coordinates
$z_e$	Factor for DNV rules
$\kappa_n$	Eigenvalue term
$\pi$	Pii
$\rho$	Density
$\varphi_{i,j}$	The natural modes

## **Abbreviations**

ABS	American bureau of Shipping
BV	Bureau Veritas
CFD	Computational fluid dynamics
DNV	Det Norske Veritas
FEM	Finite element method
FPSO	Floating Production Storage and Offloading
LR	Lloyd's Register

# 1 INTRODUCTION

## 1.1 General

Oil production started on the coastal waters around the 1890's. The first submerged platform was built on a seabed. Between the platforms, pipelines were built where oil was transported. For that reason, platforms don't need storage capacity. Oil production has been moving into deep-water because new oil resources are located in deep-water areas and demand for oil is growing all the time. This means that new production facilities need to be build on seabed. New technology offers many opportunities for deep-water oil production which are FPSO, FSO, spar etc [1]. FPSO is a floating, production, storage and offloading vessel. FPSO is a link between oil production, storage and an oil tanker. Figure 1 shows one of the possibilities how the FPSO vessel operates at sea. A FPSO vessel collects oil from nearby drilling platforms and completes underwater wells. The next step is oil processing and after that oil is stored in FPSO vessel oil tanks or tanker-offloading buoys. Another possibility is that the FPSO vessel collects oil directly from the seabed and stores the oil own tanks. FPSO offers a cost efficient solution to produce and storage oil in a deep-water area. Storage capacity is an important attribute in a FPSO vessel because pumped oil must be placed into storage before the shuttle tankers can transport oil to a harbour. Numbers of FPSO vessels increase all the time and the FPSO vessel share for future floating offshore installation projects is 60 percent. 70 percent of that share is conversion projects and 30 percent is new-builds [2][3].

New technology has brought new challenges for designers. A FPSO vessel consists of four parts which are hull, turrent, topside structure and deck house. Major part of the FPSO vessel's hull is made from tank structure. Fatigue capacity is an important issue in FPSO hull structure. FPSO vessels hull affects many other loads and sloshing is one of them. Sloshing means that the free liquid surface begins to move in a tank. Sloshing causes rapid impulse pressure in bulkheads and pressure



Purpose of first target is to understand the sloshing loads in FPSO tank structure and how significant sloshing is in FPSO tank design in concept design phase. The purpose of the second target is to study different way to calculate sloshing loads and select the most suitable procedure for design.

After this thesis the calculation procedure for sloshing loads in concept design phase is developed. Also the knowledge about sloshing pressure calculation is better and we can say when sloshing is governing load in tank and how we can decrease sloshing loads in the tank.

### **1.3 Limitations and assumptions**

The scope of this thesis is sloshing loads in FPSO cargo tanks structure design in concept design phase. FPSO vessel's other tanks such as ballast water tanks are out form the study [4].

Tank geometry is limited to a rectangular tank as it is susceptible for high sloshing loads. In addition, rectangular tanks are commonly used to fluid tank in the industry [5]. The loads are calculated by using the worst case tank acceleration so that tank location does not affect the analysis. In this thesis the natural period of the vessel is calculated for the full load case.

In vessel motion analysis the water is smooth and can be modelled with potential theory. The vessel is rigid body and wetted surface is constant [6].

In this thesis sloshing pressure calculated by using methods suitable for concept design stage.

## **2 Sloshing in FPSO tank**

### **2.1 FPSO**

The first oil FPSO vessel was the Shell Castellon built in Spain in 1977 [7]. Today over 200 FPSO vessels are operating around the world because FPSO is an economical solution in deep-water oil production. A FPSO vessel provides a large storage capacity. Once the oil has been processed it can easily be moved to an oil field.

A FPSO vessel may be built in two ways. The first way is to new build a FPSO vessel. New build solutions are costly and take a long time to complete. New build costs approximately 100-200 million dollars. A vessel's price depends on the size and complexity of the topside structure. A new build FPSO vessel project may take 3-4 years to complete. The second option is a tanker conversion which is a compromise between design and new build advantages. Selection depends on the following factors: economics, field life, residual value of the used system and possibility for redeployment. The client needs to optimise the advantages. Usually economics are the key factor in decision making. A conversion tanker is an economical option if design life is 5-15 years. When the design life is more than 20 years, new build is a better option.

Some advantages of new build FPSO vessels are:

- Design and fatigue life for an oil field can be optimized easier
- Resale value and reusability options can be improved
- Technical, commercial and environmental risks are more easily minimized
- Systems are easier to design to survive harsh environments

The advantages of a tanker conversion are as follows:

- Capital costs are lower

- Design and construction schedule can be faster and limited
- Availability of construction facilities has increased
- Overall project supervision requirements can be less

A FPSO vessel's ship resistance is not necessarily an important factor in design because the main purpose of the vessel is not to transport cargo. This enables the use of different types hull shapes. A typical barge FPSO vessel is presented in Figure 2-1. The shape is like a box ship. Usually these types of vessels are operating in a sea area where environmental conditions are mild. Hydrodynamic loads are important, because the hydrodynamic forces affect a vessel's motion. The barge hull shape is not suitable for harsh sea conditions. In harsh conditions, the hull shape should be more like a ship. This type of a FPSO vessel is presented in figure 2-2. Figure 2-1 shows that the barge type FPSO vessel's storage capacity is maximised because the prismatic coefficient is greater than in a normal tanker [8].



**Figure 2-1 A barge type FPSO is feasible for mild environmental conditions. Vessels name is Dalia [39].**

In a tanker conversion the hull modifications are minimal. Thus, no changes are made to the hull framing, plate thicknesses and welds when converted to FPSO. This is only possible if the tankers hull structure has sufficient residual strength, buckling and fatigue capacities to operate in the specified environment without dry docking during the proposed period of time. The tankers previous operating history and used production standards should be taken into account when choosing a suitable tanker for conversion [9].

A typical tanker conversion FPSO vessel is presented in Figure 2-2.





**Figure 2-2 A barge type FPSO is feasible for harsh environmental conditions. Vessels name is Peregrino [40].**

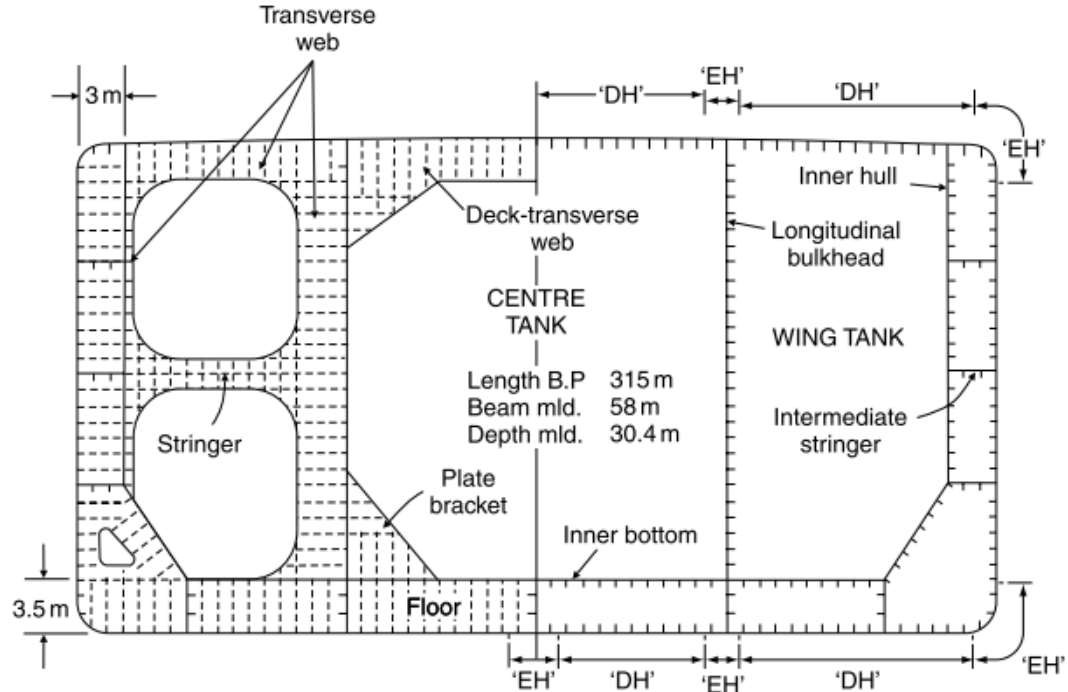
The topside structure is above the hull. The hull consists of three parts which are bow, mid-body with the tank structure and stern.

### **2.1.1 FPSO tank structure**

The FPSO tank arrangement determines the vessel's cargo capacity. The number of tanks is an important parameter from operational and cost points of view. Adding more tanks usually means higher production, operation and maintenance costs [8] and more equipment. On the other hand, storage capacity defines the shuttle tankers size and the frequency of their trips [10].

Tank structure is a large part of the hull. FPSO's tank structure is quite similar to that of an oil tanker. The structure includes bottom, shell plating, framing, pillars, bulkhead and tank top; see figure 2-1. A double bottom is not mandatory for a FPSO vessel. Usually a FPSO vessel is an old tanker where a double bottom is mandatory. Longitudinal bulkheads and transverse bulkheads divide the tank. Longitudinal

framing is an important feature so that continuity of strength is maintained, particularly in the longitudinal bulkheads which form the ends of the tanks.



**Figure 2-3 Typical tank structure [41].**

Stiffened transverse structures support the longitudinal parts, side shell and bottom structures. Transverse structures include a transverse web frames and a transverse bulkheads. Plate bracket reduces the peak pressure on the corner. The floor structure includes bottom, inner bottom and keel. A stiffened bottom and a stiffened inner bottom combine are combined by the keels [11].

### 2.1.2 Loads in the FPSO tanks

The vessel tank structures are exposed to many different load types. Load types can be divided into static loads and dynamic loads. Static loads contain all still water loads. Those loads are external and internal pressures. Dynamic loads contain slowly varying loads and rapidly varying loads. Slowly varying loads contain sloshing loads and the wave-induced dynamic pressure. Sloshing loads should be derived

from ship motion analysis but computation is difficult because sloshing is a highly nonlinear phenomenon [12].

Classification societies divide tank structure design loads into three different categories. Those categories are hydrostatic pressure; hydrodynamic pressure; and sloshing and impact pressure. Sloshing and impact pressure are in the same category as some of classification societies treat them simultaneously (ABS) and others separately (DNV) [13][14].

The tank structure pressure equation can be divided into three parts.

$$P_{TOT} = P_{static} + P_{sloshing} + P_{hydrodynamic} \quad (2-1)$$

where  $P_{TOT}$  is total pressure inside the tank,  $P_{static}$  is static,  $P_{sloshing}$  the sloshing and  $P_{hydrodynamic}$  is hydrodynamic pressure inside tank.

### **2.1.3 New build tank structure vs. conversion tanks structure**

Typical FPSO new build hull structure is single bottom and double sided. Offshore classification rule allows the single bottom hull because damage to the bottom is unlikely in deep-water areas. The single side is also an accepted solution but a FPSO vessel needs a ballast water capacity. The double side also protects cargo tanks from damage caused by collisions with shuttle tankers. Possible dangerous situations are; shuttle tanker and FPSO are side by side, passing a vessel or supply boats collides to the FPSO vessel [8].

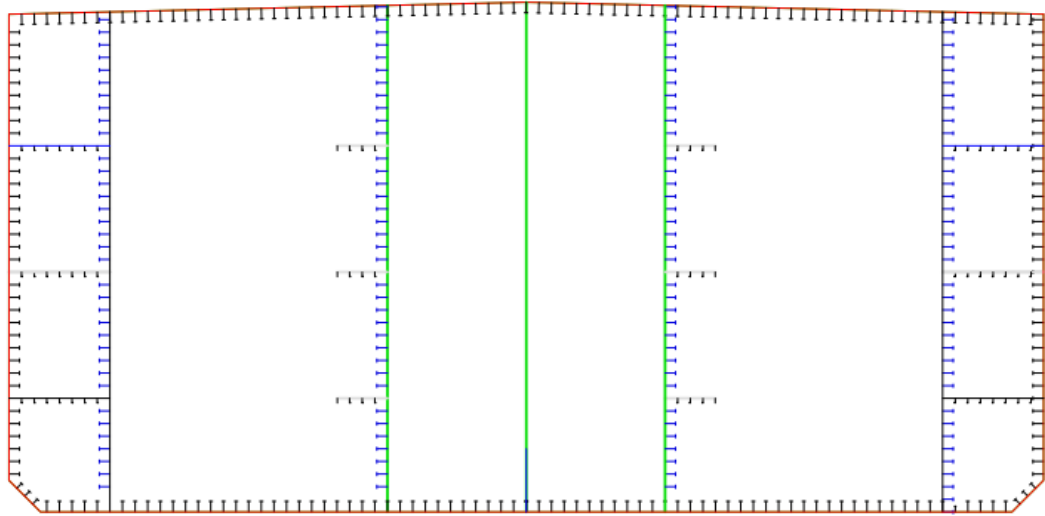
Designing the new build tank arrangement and hull, there are a few issues which need to be taken into account in the design process. Those relevant issues are listed below:

- Number of cargo and ballast tanks.
- Location of cargo and ballast tanks.
- Size of cargo and ballast tanks.

- Tank strength, corrosion protection and access.
- Pumping arrangement for tanks.
- Location and size of tanks required for special services such as slop tanks, chemical tanks etc.

Each of the above conditions needs to be considered for the hull design and tank general arrangements [8]. The first three issues describe the tank capacity. The tank strength issue describes tank structure sustainability. The pumping systems describe how liquid moves.

The new build FPSO has several cargo tanks located centrally and several ballast water wing tanks arranged on either side. Usually ballast water tanks are located inside the double sides. Example of a new build tank structure cross section is presented in Figure 2-4.



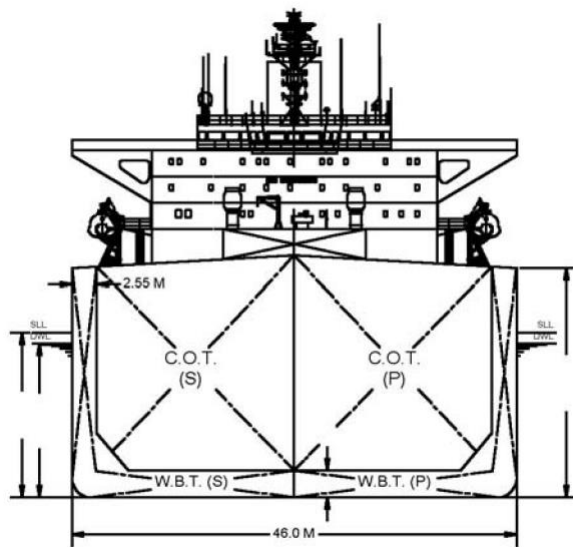
**Figure 2-4 Example of a new build FPSO tank structures cross section.**

The bottom is a stiffened single bottom. Structures are supported with longitudinal bulkheads which divide the tank area into cargo tanks and ballast water tanks. Longitudinal bulkheads make it easy to design a topside structure, because these bulkheads offer good support points. Between the double sides are horizontal stringers

that support the side structures so that the cargo pressure will not cause the sides to collapse. For harsh environmental conditions, the FPSO cross section is presented in figure 2-5. The figure shows that there is a double bottom and double sides.

Conversion FPSO hull structure is usually double-skinned, because currently every classified tanker must have a double hull structure. The double bottom is not removed because usually a double bottom is one part of a complex tank structure. A double bottom carries out lot of different loads and the structure will fail if the inner bottom is removed.

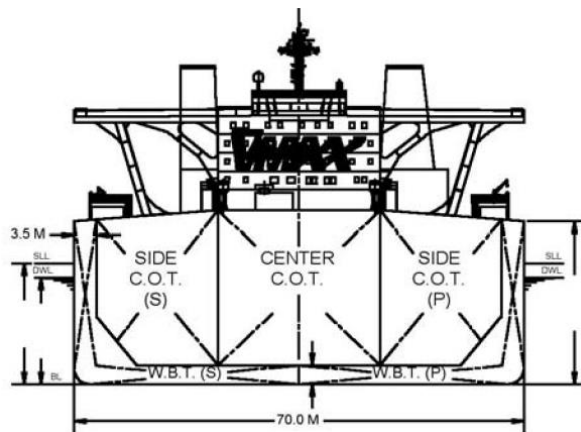
Figure 2-5 presents amidships section of a basic tanker where two tanks are in the transverse direction. This kind of a tank structure is weak in the topside design because the supporting structures are less stiff and there are no hard points. A hard point is an intersection of the transverse and longitudinal bulkheads.



**Figure 2-5 M/T British Harrier cros section of tank structure [42].**

Figure 2-5 shows how the inner bottom skin is a large part of the continuous structure. Smooth inner bottom will cause the bottom structure not to dampen sloshing effect.

In figure 2-6 the other typical tankers amidship section is presented. Stena Vision has three tanks in the transverse direction. Tanker breadth is large so the tank needs to split into three parts. This kind of tank structure is an excellent FPSO conversion target because the structure offers many strong structural points. The tank structure is quite similar to that in figure 2-3 structure but the plate brackets are larger because the tank pressure is excessive on the corner.



**Figure 2-6 M/T Stena Vision cross section of the tank structure [42].**

Figures 2-5 and 2-6 show that tank sizes are quite similar even though ship breadth is larger. Therefore, the tank area is divided into two or three parts in longitudinal direction. Conversion tanker tank structure is optimized for tanker use. Converting a tanker into FPSO may cause sloshing problems because tanker tank structure is designed for fully loaded or empty tanks. Full load and empty means that possible design loads are calculated for full or empty tanks. In FPSO vessel the liquid varies in the tanks. This means that possible design loads need to calculate different filling levels.

## **2.2 Sloshing**

### **2.2.1 Sloshing phenomenon**

Sloshing is a common phenomenon that occurs in a moving vehicle or structure containing a liquid with a free surface. A free liquid surface is located in a closed tank. Structure is subjected to an external force which causes an impulse in structures. The impulse affects a closed tank where the liquid free surface start to oscillate. Usually sloshing can be the result of resonant excitation of the tank liquid. Sloshing can also be transient motion [15]. The behaviour of the liquid in a tank can be violent if the excitation frequency is close to the tanks natural frequency [16].

Sloshing is a strong non-linear phenomenon which can be divided into two parts. First, sloshing is a large scale global phenomenon which occurs because of global flow inside the tank and the restriction on the movements of the ship, the tank geometry and the liquid level. Secondly, sloshing is a small-scale phenomenon which creates local pressure loads across the tank structure [17].

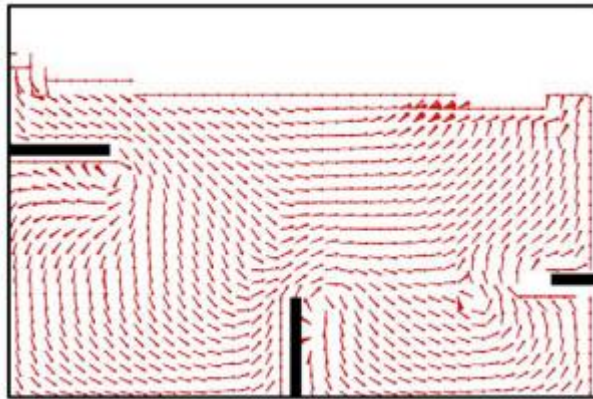
Sloshing phenomena studies can be classified as analytical, experimental and numerical studies. The analytical method is mostly limited to rectangular tanks. In the experimental method, impulsive pressure is determined in a model test. A model test is carried out to closely simulate the six degrees of free motions of a ship. The numerical analysis method is one of the more popular methods in simulating a non-linear free-surface behaviour and these can estimate the magnitude and location of the impulsive pressure acting on the tank walls [18][19][20].

Global forces and moments are important but more important in most ships are local impact pressure because sloshing can cause critical damage in a ships tank. Sloshing in a ships tank is difficult phenomena because the resonance between ship motion and natural mode of fluid motion is hard to avoid [21].

Sloshing is very difficult to control but there are different ways how to minimize the sloshing loads. One way is to create some damping effect. Damping effect means that the liquid mass does not receive a high acceleration amplitude inside the tank.

### 2.2.2 Sloshing damping

Damping of sloshing has an important role when sloshing is a relevant phenomenon on tank structure. Damping causes acting sloshing pressure on the wall to decrease. Typical structural dampers are swash bulkheads or plates. The plate may be in vertical or horizontal direction. Figure 2-7 shows how the plate may be placed inside the tank. A swash bulkhead is a plate with lightening holes.

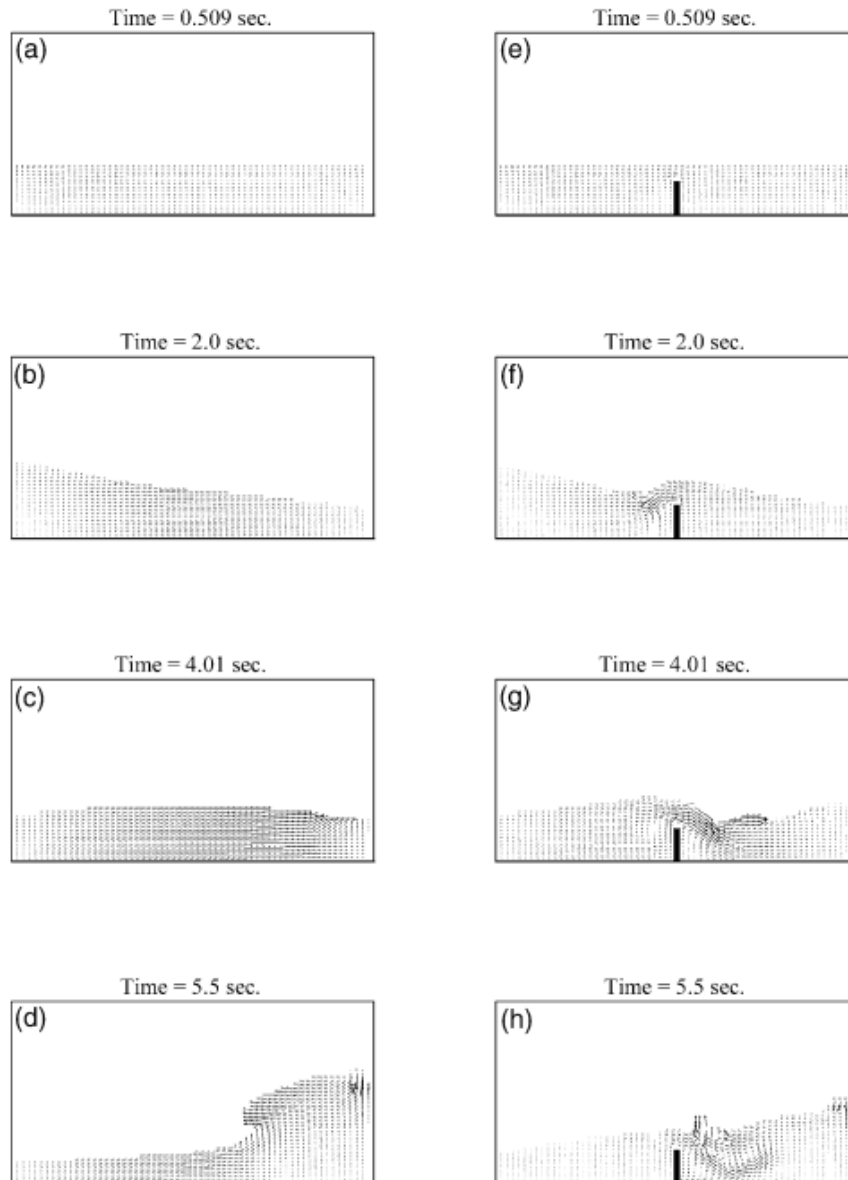


**Figure 2-7 Damping plates in horizontal and vertical direction [43].**

Figure 2-8 shows how the vertical plate effects tank centre damping sloshing flows. The figures left side is a normal tank where there isn't any flow damping and fluid moves freely causing high pressure to the tank wall. The figures right side is a normal tank where there is a vertical plate in the middle of the tank. The purpose of the vertical plate at tank centre is to break the sloshing flow and decrease the effective sloshing pressure on the tank wall. The research showed that the effect of the verti-



cal plate was most predominant when the tank water level is low. Flow over a vertical plate produced a shear layer which received energy from the flow. For this reason, the overall turning moment decreased significantly [22].



**Figure 2-8 Sloshing flow with and without vertical plate [44].**

Frequency of the tank with a vertical baffle affected directly the degree of non-linearity of the sloshing phenomenon. Other parameters are fill depth and rolling

amplitude [22]. Vertical baffle damping effect is increased when width is the primary tank length.

In concept design phase the best way to calculate sloshing loads is to use classification society's rules and to focus on main dimensions which control the free surface. Each classification society's define own rules. CFD calculations are also suitable for research work and detail design, but not for concept design due to their large computational cost.

### **2.2.3 Sloshing in FPSO tank**

In a FPSO tank the amount of oil varies constantly because oil is pumped from a sea bed to a FPSO tanks and from a FPSO tank to a shuttle oil tanker. This will cause the liquid level to vary in the tank and so, sloshing can occur in the tank. Sloshing phenomenon occurs in FPSO tanks when oil in the tank begins to resonate. Resonance occurs when the ships natural motion is near the natural modes of fluid motion. FPSO tank sloshing is to be expected for all load conditions because head or nearly head sea is dominant. Particularly pitch motion is important for sloshing.

Sloshing loads affect many different factors and those factors can be divided into three parts. Those parts are tank dimensions, tanks structural arrangement and fluid motion. The tanks dimensions include filling level and tank dimensions. Structural arrangements includes all parts inside the tanks. The fluid motion includes; the longitudinal and transverse metacentric height GM, natural periods of cargo and unit in pitch and roll modes [4].

Tank dimensions height  $h$ , length  $l$  and breadth  $b$  define the volume of the tank. Significant tank dimensions of sloshing loads are effective length of the tank  $l_e$  and effective breadth of the tank  $b_e$ . Effective length and breadth means the tank has a free length without any damping effect. If inside the tank there is some damping, it decreases the tank effective length and breadth. Tank height is not an important

parameter for sloshing but filling level is. Usually a small filling level causes the highest rapid pressure to the tank wall [4].

Structural arrangements inside the tanks are one large assembly, which affects the sloshing loads. Parts inside the tank dampen and slow down the fluid motion. Those parts are longitudinal and transverse wash bulkheads, web frames, longitudinal and transverse girders. Girders may be located in the tank bottom or sides. If the tank has a double bottom and double sides, the surface is smooth and does not dampen the fluid motion. In this case, transverse and longitudinal bulkheads need to take the sloshing peak pressure [4].

Ship roll and pitch motion also affect sloshing pressure because ship motions causes the fluid motion in the tank. Resonance between ship motion and fluid motion is to be expected because fluid height varies in tanks. GM affects longitudinal bulkheads and high GM causes greater sloshing pressure. Factors affected by metacentric height are the vertical distance between the vessel's centre of gravity and the baseline  $KG$ , the displacement between the centre of the gravity height from the baseline  $KB_0$  and metacentric radius  $B_0M_0$ . The hulls shape affects the magnitude of those factors. Increasing the value of  $KG$ , decreases the value of  $GM$ . Increasing the values of  $KB_0$  and  $B_0M_0$  increases the value of  $GM$  [23].

## **2.3 Rule approaches for sloshing loads**

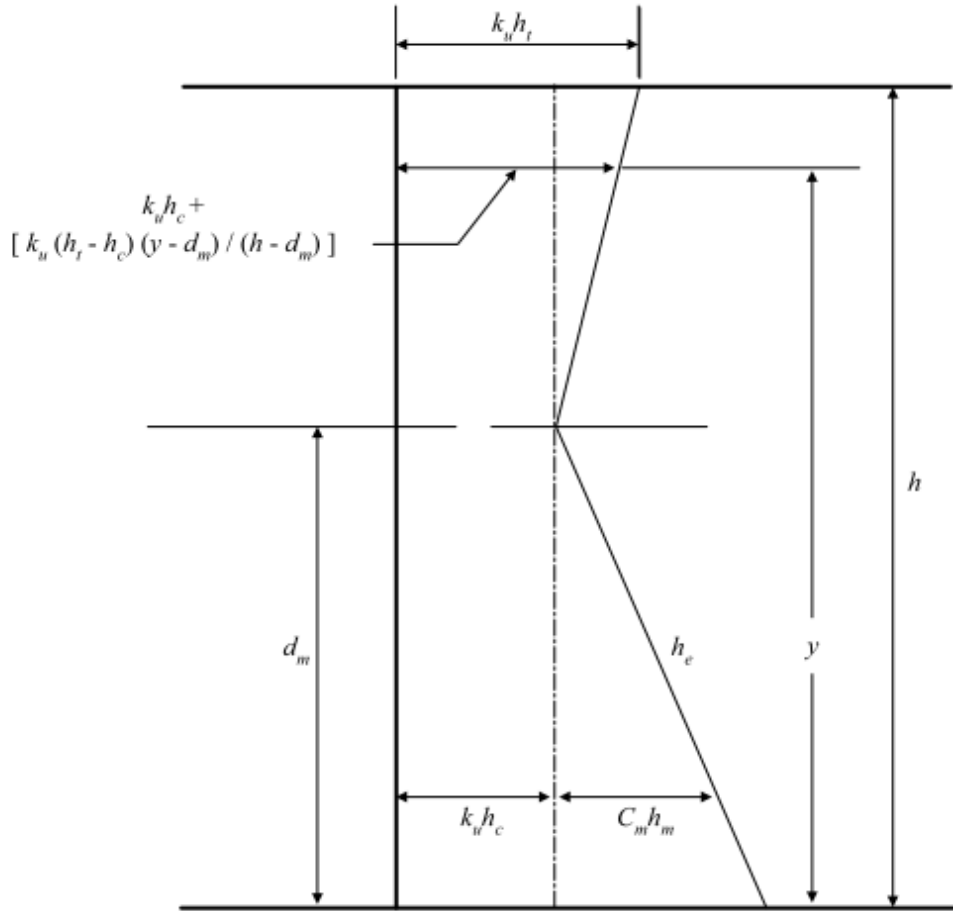
All major IACS members have a classification service for offshore units and American Bureau of Shipping, Det Norske Veritas, Bureau Veritas and Lloyds Register of Shipping have rules and standards for FPSOs [24]. Every classification society provides rules for sloshing pressure calculations. In partially filled tanks, the probability increases that the fluids natural period and the ships natural period are the same.

The background for offshore sloshing pressure rules are tankers sloshing rules. One reason for this is the fact that many of the FPSO vessels are conversion of old tankers. In sloshing pressure calculations DNV, BV and LR offshore rules refer to ship rules. The ship rules are the IACS common structural rules for double hull oil tanker. IACS rules define that sloshing pressure calculated for filling heights of  $0,05h_{max}$  to  $0,95h_{max}$  in increments of  $0,05h_{max}$  [25]. ABS has developed its own sloshing rules for FPSO sloshing load calculations that deviate from rules set by IACS. ABS offshore rules define that sloshing pressure calculated for filling heights of  $0,20h_{max}$  to  $0,90h_{max}$ .

### **2.3.1 American Bureau of Shipping sloshing rules**

ABS classification of offshore rules are the most extensive for sloshing loads. Sloshing pressure calculation is required for the FPSO vessels. A sloshing calculation procedure constitutes of a grid with 27 nodes around the tank wall. This means that sloshing pressure can be defined at each point of the tank. The rules basic idea is to combine sloshing pressure and hydrostatic pressure. Sloshing pressure also includes possible impact pressure which occurs inside the tank. When the calculation point is above the filling level, the point only affects sloshing pressure. The hydrostatic pressure part is added when the calculation point is below the filling level. The design pressure for a tank structure bulkhead is sloshing pressure or internal pressure. Internal pressure means hydrodynamic pressure. The design pressure depends on whichever is larger. Figure 2-9 presents a vertical distribution of

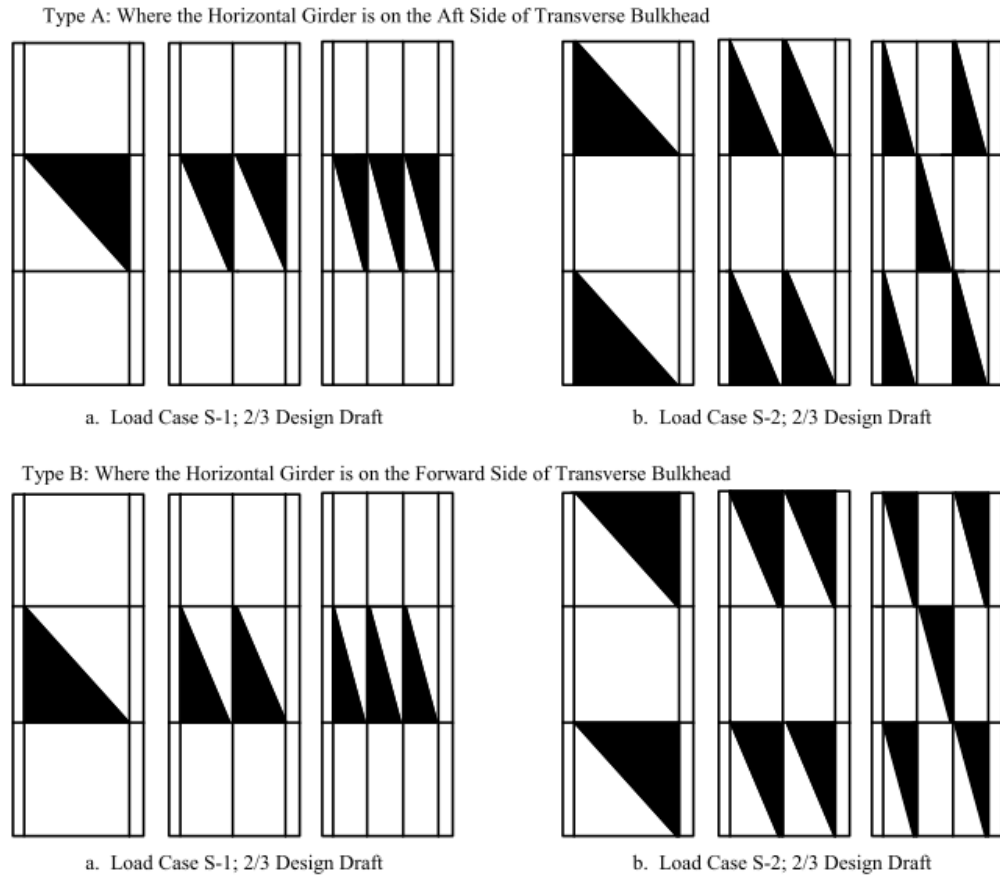
the sloshing pressure. Pressure is linear from bottom to filling height and from filling height to top of the tank. Lowest pressure is achieved at filling height. Sloshing pressure is high in the upper part when the filling level is low. On the other hand, hydrostatic pressure is greater in the lower part when the filling level increases. Sloshing pressure has a bilinear trend- while hydrostatic pressure has linear trend when the sloshing pressure standardize is connected to the hydrostatic pressure.



**Figure 2-9 Vertical distribution of equivalent slosh pressure head [45].**

Figure 2-10 present the different load patterns for sloshing load cases. The rule has two different load cases, one for the horizontal girder on the back side of the transverse bulkheads and two load cases for the horizontal girder on the front side of the

transverse bulkhead. Load patterns depend on how many tanks are in the vertical direction. Usually there are two or three tanks but sometimes only one large tank. Figure 2-10 shows that load patterns are given as a triangle and peak pressure is focused on top point of the triangle. The triangles direction depends on the ship's motion.



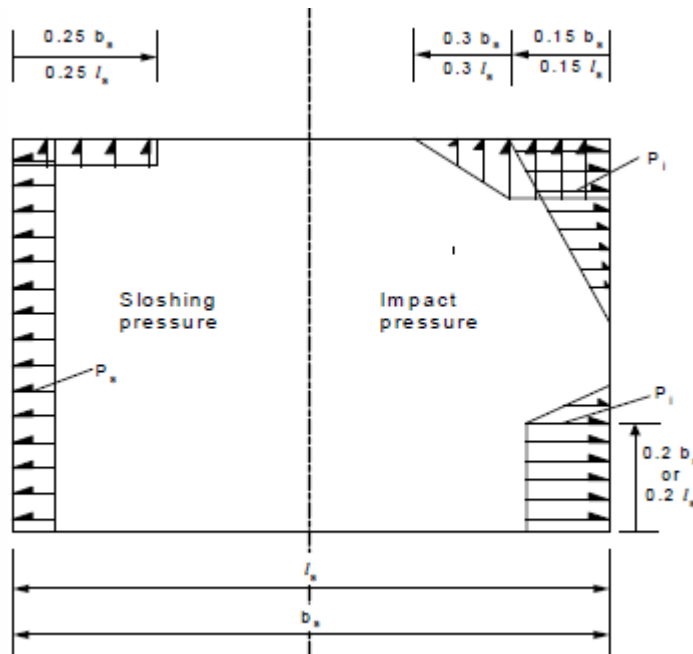
**Figure 2-10 Loading patterns for sloshing load cases [45].**

### 2.3.2 IACS sloshing rule

IACS sloshing rule is the basis for the other major classification society sloshing rules. DNV's and LR's ship rules on the sloshing part are the same as IACS sloshing part [26][27][28][4][14][29][30]. Bureau Veritas offshore rules describes sloshing

phenomena and the risk of resonance. Offshore rules refer to ship rules in sloshing pressure calculations [31].

IACS's sloshing rules are quite simple if we compare ABS's rules. IACS's sloshing pressures distribution is presented in figure 2-11. Sloshing pressure is divided into two parts which are sloshing pressure and impact pressure. Impact pressure in the upper part of the tanks and lower part of the tanks is calculated if some conditions are feasible. For example impact pressure in the upper part of the tanks calculated if tanks with free sloshing length is  $0.13L < l_s < 0.16L$  or with free sloshing breadth  $b_s > 0.56B$ .



**Figure 2-11 DNV sloshing pressure distribution in tank [46]**

Minimum sloshing pressure in a cargo tank is  $20 \text{ kN/m}^2$  and in a ballast tank it is  $12 \text{ kN/m}^2$ . Sloshing pressure may be reduced by increasing the wash bulkhead, transverse web-ring or longitudinal ring girders.

Sloshing pressure applied to transverse bulkheads can be calculated according from.

$$p = \rho g_0 l_s k_f [0,4 - (0,39 - \frac{1,7 l_s}{L}) \frac{L}{350}] \quad (2-2)$$

where  $\rho$  is density of liquid in  $t/m^3$  which should not to be less than 1,025  $t/m^3$ ,  $g_0$  is gravitational acceleration in  $m/s^2$ ,  $l_s$  is the effective sloshing length in meters,  $k_f$  is a factor and  $L$  is the ship length in meters.

Sloshing pressure applied to longitudinal bulkheads can be calculated according from.

$$p = 7\rho g_0 k_f (\frac{b_s}{B}) GM^{0,75} \quad (2-3)$$

where  $b_s$  is the effective sloshing breadth in meters,  $B$  is the ships breadth in meters and  $GM$  is maximum  $GM$  including a correction for the free surface effect in meters. The minimum  $GM$  is  $0,12B$ . Factor  $k_f$  can be calculated according from

$$k_f = 1 - 2(0,7 - \frac{h}{H})^2 \quad (2-4)$$

where  $h$  is the filling height in meters and  $H$  is the tank height in meters.

IACS sloshing pressure calculation gives a fast result and thus is suitable for concept design. However, the sloshing calculation procedure is simplified and therefore less accurate. The problem is that only the filling level changes and the pressure value doesn't change along the bulkheads width. The calculation procedure doesn't take into account the fluids natural period or the vessel's natural period. One reason for this may be that this sloshing calculation procedure was aimed at tankers. In the tanker's tanks the filling level doesn't vary and the load cases are either a full load or an empty tank. The second problem is that the impact pressure in upper part of tank is valid only when sloshing length is in the range of vessel length. Thus, the sloshing pressure calculation is less accurate than ABS sloshing results.



### 3 Analysis Procedure

#### 3.1 Liquid natural period

Liquid's natural period and frequency varies when a liquid's filling level changes in the tank. The lowest natural frequency tells when sloshing is to be expected in a tank [15]. The liquid resonance frequency varies for different excitation amplitudes, tank structure arrangements, liquid densities and viscosities [32]. These are all non-linear problems and the resonance does not occur exactly at the natural frequency but at a frequency very close to that value [33].

Faltinsen has developed mathematical model, which calculates natural sloshing modes and frequencies. The model assumes linearized conditions with no tank excitation. The free surface boundary conditions are linearized. The eigenvalue  $\kappa$  can be solved by using spectral boundary problem analysis. In a two dimensional case, the boundary conditions lead to the general solution for  $\kappa$ . The eigenvalue term in two dimensional rectangular tanks can be calculated according from

$$\kappa_i = \frac{\pi^i}{l} \tanh\left(\frac{\pi^i}{l} h\right) \quad i \geq 1 \quad (3-1)$$

where  $l$  is the tank length in meters and  $h$  is the liquid depth from the bottom to the surface in meters. The natural sloshing periods and frequencies can be calculated according to 3-2 and 3-3. The equations show that gravitational acceleration, tank length and liquid depth affect natural sloshing periods and frequencies.

$$T_n = 2\pi / \sqrt{g\kappa_n} \quad (3-2)$$

$$f_n = \sqrt{g\kappa_n} \quad (3-3)$$

where  $\kappa_n$  is eigenvalues term and  $g$  is gravitation acceleration. Equation 3-2 can be combined with equation 3-2 and 3-3. Now we can calculate the natural sloshing periods and frequency according to 3-4 and 3-5

$$T_i = \frac{2\pi}{\sqrt{g\pi i \tanh(\pi i h/l)/l}} \quad i = 1, 2, \dots \quad (3-4)$$

$$f_i = \sqrt{g \frac{\pi i}{l} \tanh\left(\frac{\pi i}{l} h\right)} \quad i = 1, 2, \dots \quad (3-5)$$

The first mode of natural sloshing period can also be estimated from figure 3-1. The figure shows a two dimensional rectangular tanks first mode of natural sloshing period. Figure's results are relevant for ship tanks. The natural period decrease when tank length or fluid depth increase.

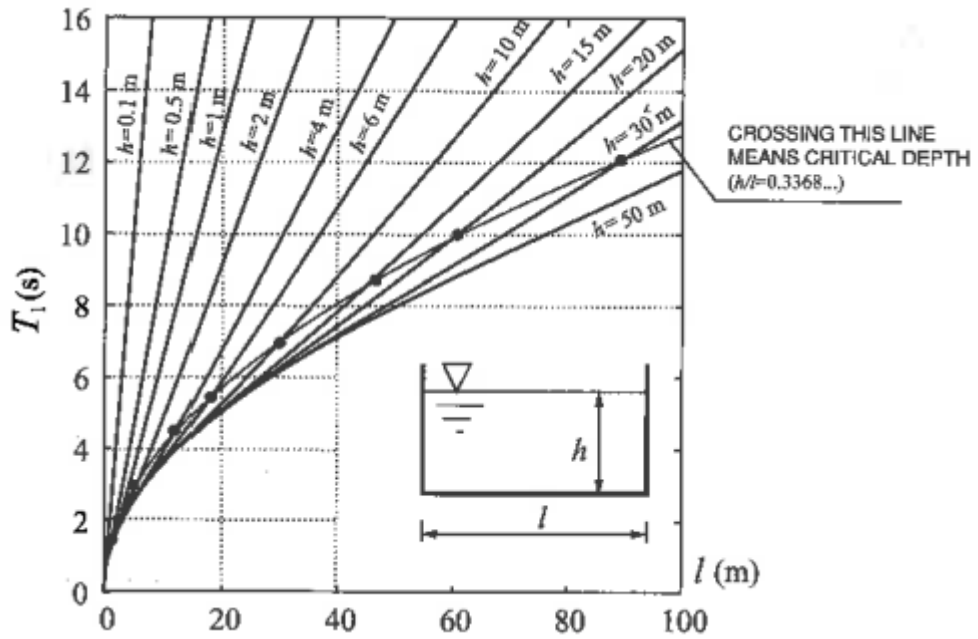


Figure 3-1 First mode natural sloshing period for a two dimensional rectangular tank [47].

ABS has its own system for calculating the fluid natural periods. The natural period of the fluid motion in longitudinal- and transverse directions can be calculated according to equations 3-6 and 3-7.

$$T_x = \frac{(\beta_T l_e)^{1/2}}{k} \quad (3-6)$$

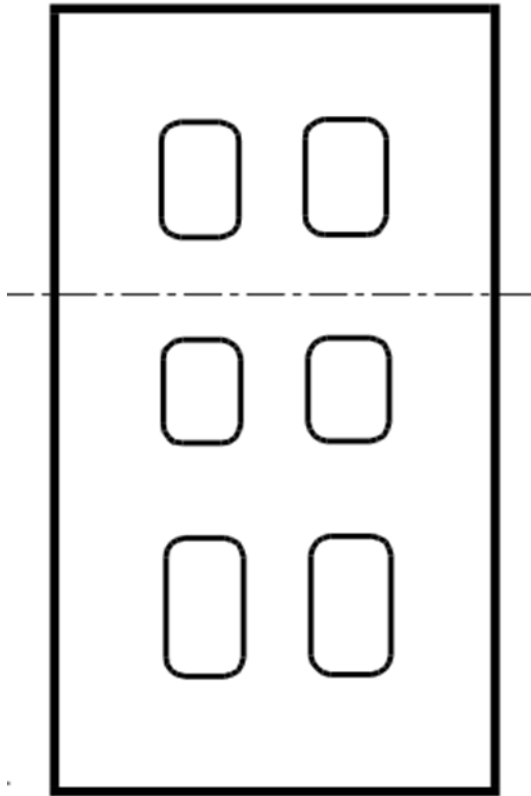
$$T_y = \frac{(\beta_L b_e)^{1/2}}{k} \quad (3-7)$$

where  $l_e$  is effective length of the tank in meters,  $b_e$  is effective length of the tank in meters and  $k$  can be calculated according from

$$k = \left[ \frac{\tanh H_1}{4\pi/g} \right]^{1/2} \quad (3-8)$$

$$H_1 = \frac{\pi d_l}{l_e} \text{ or } \frac{\pi d_b}{b_e} \quad (3-9)$$

where  $d_l$  and  $d_b$  are filling level values. Those values take into account possible damping effects in transverse- and longitudinal directions. The full swash bulkhead which used in the case study vessel, is presented in figure 3-2. In this case study a single type of swash bulkhead is used because generally a tanker swash bulkhead is full swash.



**Figure 3-2 Full swash bulkhead [45].**

Sometimes second order resonance can occur when the sum of the frequencies or the difference frequency on any two excitation components is equal to one of the natural frequencies [34].

The general form for higher modes and expected secondary resonance for the N th mode can be estimate according 3-10 [15].

$$\frac{f_1}{f} = \frac{T}{T_1} \approx i_n = \frac{nT_n}{T_1} = \sqrt{\frac{n \tanh(\pi h/l)}{\tanh(n\pi h/l)}} \quad (3-10)$$

By calculating the natural period of the fluid, it is know when the liquid will starts to oscillate in the vessel's tanks. This information is important in tank design because the fluid and the tank natural periods can't be the same. The other important factor which affects the size of the sloshing load is fluid acceleration. The fluid acceleration is easy to calculate by using the vessel's motion analysis.

### 3.2 Motion analysis of FPSO

Ship motion affects the fluid in the cargo tanks. The free surface fluid motion depends on the ship motion. By solving the motion in the vessel's tank, the acceleration of the fluid in the tank can be defined. The assumption is that fluids acceleration equals the tank's acceleration. The vessel's tank acceleration can be calculated by Ansys AQWA LINE program.

Ansys AQWA LINE-program solves the diffraction- radiation- and Froude-Krylov forces. The diffraction force describes the forces which are due to wave diffraction at the motionless hull surface. The radiation force describes the force which acts on the oscillating hull in still water conditions. The Froude-Krylov force describes the pressure variation due to wave height.

The motions are solved by using.

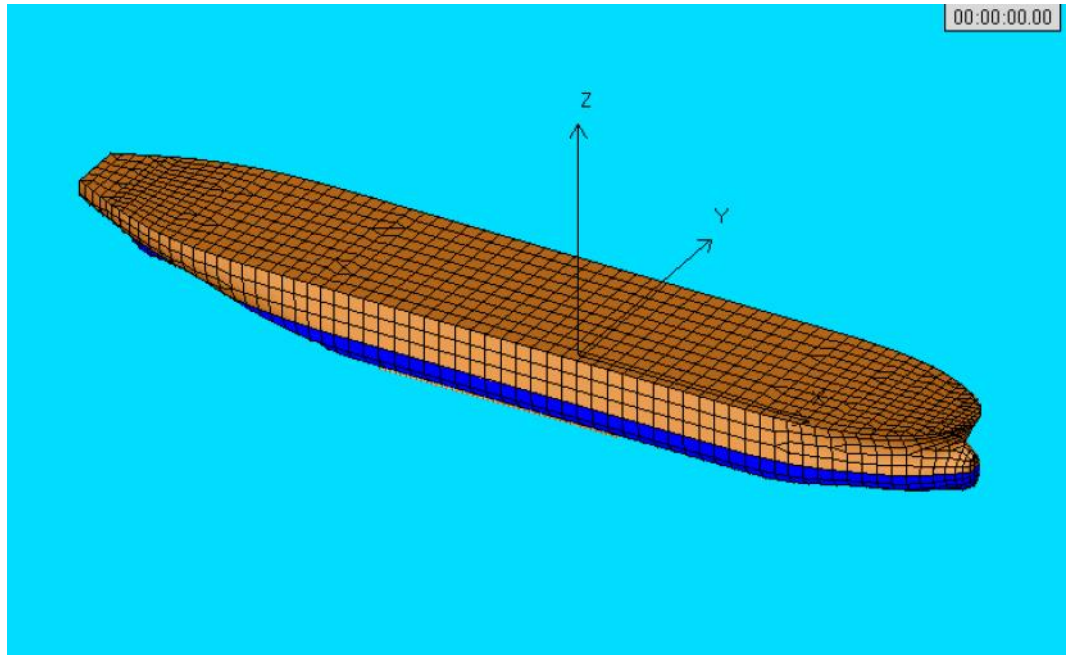
$$F = M(s)\ddot{X} + M(a)\ddot{X} + C\ddot{X} + K(s)X \quad (3-11)$$

$$F = F_0 e^{-i\omega t} \quad (3-12)$$

$$X = X_0 e^{-i\omega t} \quad (3-13)$$

where  $M(s)$  is the structural mass matrix,  $M(a)$  is the hydrodynamic added mass matrix,  $C$  is the system linear damping matrix,  $K(s)$  is the total system stiffness matrix,  $X$  is response to motion and  $F$  is the external wave forces on the system and  $\omega$  is the frequency of wave forcing. A mass distribution is needed to construct the  $M(s)$  matrix. The other parameters are hydrodynamic.

The surface of the hull structure is modeled by using panels. The middle point of every panel is called a source. Source strength is calculated assuming that there is no flow through the hull, seabed and taking into account the free surface condition. Every panel's source strength is assumed to be constant. The model coordinate system is a right handed Cartesian co-ordinate system is centre of gravity of the ship. The AQWA LINE model is showed in figure 3-3.



**Figure 3-3 Diffraction model.**

The local motion at every point on the hull can be calculated assuming rigid body motion. When the velocity, acceleration, angular velocity and angular acceleration in any point A are known, the motion at any point B can be solved using

$$\bar{v}_A = \bar{v}_B + \bar{v}_{A/B} = \bar{v}_B + \bar{\dot{\theta}} \times \bar{r}_{A/B} \quad (3-14)$$

$$\bar{a}_A = \bar{a}_B + \bar{a}_{A/B}^t + \bar{a}_{A/B}^n \quad (3-15)$$

$$\bar{a}_{A/B}^t = \bar{\ddot{\theta}} \times \bar{r}_{A/B} \quad (3-16)$$

$$\bar{a}_{A/B}^n = \bar{\dot{\theta}} \times (\bar{\dot{\theta}} \times \bar{r}_{A/B}) \quad (3-17)$$

where  $v$  is the translation velocity,  $a$  is translation acceleration,  $r$  is a spatial vector and  $\theta$  is rotational motion.

In AQWA LINE model this means that the ship motion is solved only at one point. Then the ship motion can be calculated from any point in the body where a node is. Ship motion results can be considered as accurate as the calculations have been made to the right hull shape and the correct forces are action on the hull [6][35][36][37]..

### 3.3 Sloshing pressure

Sloshing pressures are calculated by using the ABS sloshing pressure rule. Sloshing pressure includes two pressure parts. The first part is hydrostatic pressure. Hydrostatic pressure increases when the filling level increases. The second part is sloshing pressures. When the filling level is low, the tank upper structures are only affected by sloshing pressure. Sloshing pressure can be calculated according from

$$p_{is} = k_s \rho g h_e \quad (3-18)$$

where  $k_s$  is the load factor,  $\rho$  is fluid density,  $g$  is gravitational acceleration and  $h_e$  is equivalent liquid pressure head which can be calculated according from

$$h_e = k_u \left[ h_c + \frac{(h_t - h_c)(y - d_m)}{h - d_m} \right] \quad \text{for } y > d_m \quad (3-19a)$$

$$h_e = c_m h_m + k_u h_c \quad \text{for } 0,15h \leq y \leq d_m \quad (3-19b)$$

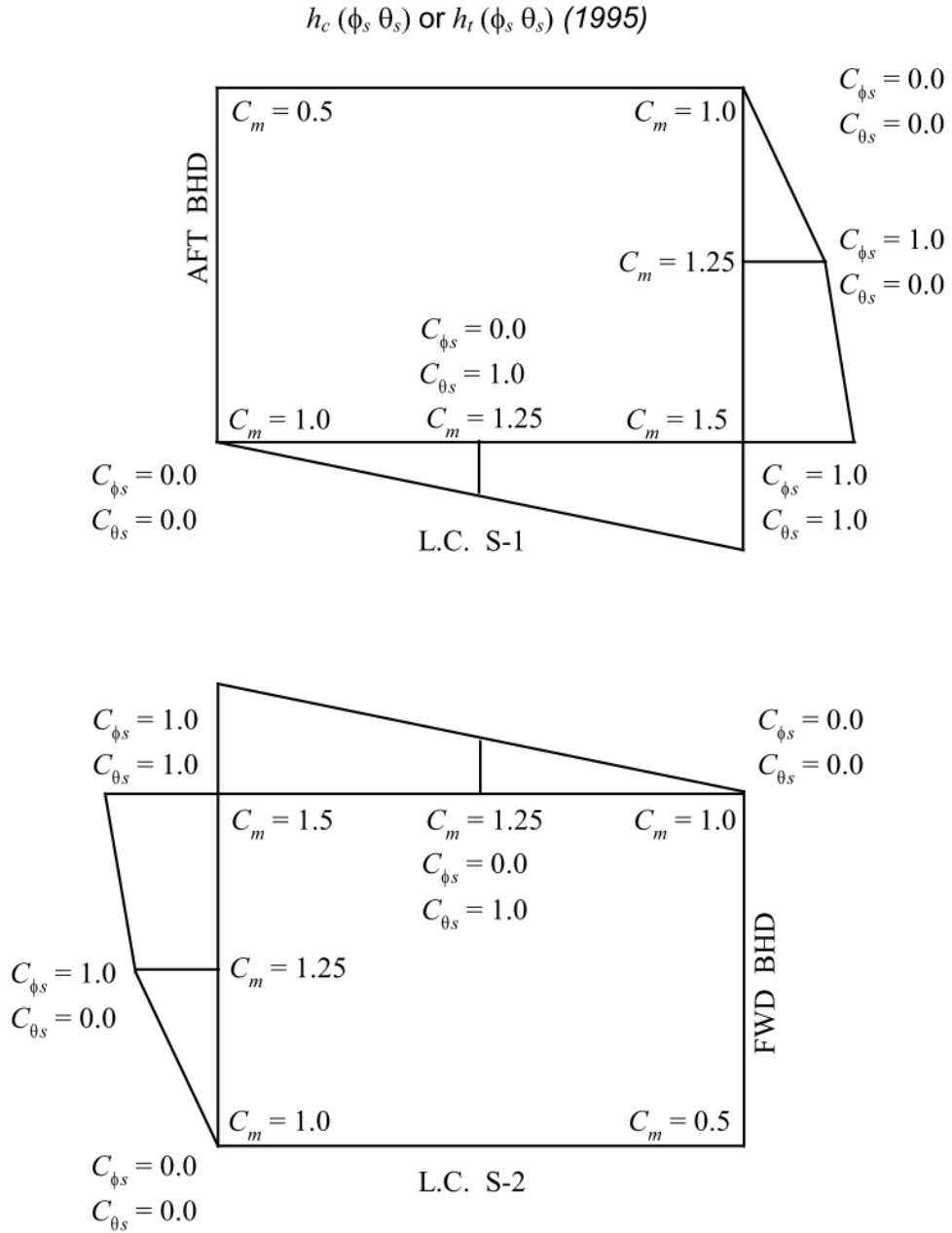
where  $k_u$  is load factor,  $h_c$  is maximum average sloshing pressure head in meters,  $h_t$  is sloshing pressure head for the upper bulkhead in meters,  $h$  is the depth of the tank in meters,  $y$  is the vertical distance from the tank bottom to the point considered in meters,  $d_m$  is the filling level in meters and  $c_m$  is the coefficient which is defined in figure 3-3. If  $y < 0,15h$   $h_e$  calculated at  $y = 0,15h$  but  $h_e$  should not be smaller than  $c_m h_m$ . The maximum average sloshing pressure head  $h_c$  and sloshing pressure head for upper bulkhead  $h_t$  can be calculated according to 3-20 and 3-21.

$$h_c = k_c (C_{\phi s} h_l^2 + C_{\theta s} h_b^2)^{1/2} \quad (3-20)$$

$$h_t = k_c (C_{\phi s} h_{tl}^2 + C_{\theta s} h_{tb}^2)^{1/2} \quad (3-21)$$

where  $k_c$  is a correlation factor for the combined load cases,  $C_{\phi s}$  and  $C_{\theta s}$  are the weighted coefficients which are defined in figure 3-4[13].

The computation code and whole procedure is presented in appendix 1.



**Figure 3-4 Horizontal Distribution of Simultaneous Sloshing Pressure Heads [45].**

Sloshing pressure may not be the dominant pressure in the tank. For that reason the hydrodynamic pressure must also be calculated when trying to look for the dominant pressure in the tank. Classification society's rule with AQWA LINE acceleration results gives better results for hydrodynamic pressure.



### 3.4 Hydrodynamic pressure

Hydrodynamic pressure is generated when the tank's acceleration starts to affect the fluid. When the fluid hits the tank's wall, the fluid causes the dynamic pressure load. The vessel's tanks hydrodynamic pressure can be calculated by using ABS classification society's rule. By default in the rules the tank is completely filled. The ABS hydrodynamic pressure results are not accurate but the results are comparable to the other pressure results. The vessel's tank hydrodynamic pressure can be calculated according to 3-22

$$p_i = k_s \rho g (\eta + k_u h_d) + p_0 \quad (3-22)$$

$$p_0 = (p_{vp} - p_n) \geq 0 \quad (3-23)$$

where  $k_s$  is the load factor,  $\rho$  is water density,  $g$  is acceleration of gravity,  $\eta$  is a local co-ordinate in the vertical direction for tank boundaries measured from top of the tanks,  $k_u$  is load factor,  $h_d$  is wave-induced internal pressure head which includes an inertial force and added pressure head,  $p_n$  is 2.06 N/cm<sup>2</sup> and  $p_{vp}$  is the pressure setting on the pressure relief valve.

Equation 3-22 includes hydrostatic and hydrodynamic pressure components. The term  $k_u h_d$  in brackets describes hydrodynamic pressure. The term  $h_d$  can be calculated from

$$h_d = k_c \left( \frac{\eta a_i}{g} + \Delta h_i \right) \quad (3-24)$$

where  $k_c$  is a correlation factor,  $a_i$  is the effective resultant acceleration in m/sec<sup>2</sup> obtained from AQUA LINE and  $\Delta h_i$  is added pressure head due to pitch and roll motion in meters. Term  $a_i$  includes surge, sway and heave acceleration components. Other equations can be found in the rules [13].

The hydrodynamic pressure effect in the tank can be a large when the fluid acceleration is large and fluid level is high. Hydrodynamic pressure is usually high in the tank structures lower part because hydrostatic pressure affects also there.

## 4 Case study

### 4.1 Case vessel

The case study vessel is a typical FPSO conversion project. The new owner wants to change the ship's type from a tanker to a FPSO. The main particulars of the vessel are given in Table 4-1.

**Table 4-1 Main particulars of the case study vessel.**

Length, overall	285.41 m
Length, between perpendicular	275.0 m
Breadth	50.0 m
Depth	22.50 m
Design draught	15.0 m
Scantling draught	16.50 m

The original tank structure arrangement is 6 tanks in the longitudinal direction and 2 tanks in the vertical direction. A conversion FPSO has internal turret which takes up two tanks in the vertical direction. Thus, the FPSO vessels tank design structure has only 5 tanks in the longitudinal direction and 2 tanks in the vertical direction. The case study vessels midship section is presented in figure 4-1 and a general arrangement is presented in figure 4-2. The case study vessel tank structure includes a double bottom and double sides. Removing the double bottom is impossible because otherwise the bottom structure would collapse. Therefore, the effect of removing the double bottom for sloshing loads is not examined.

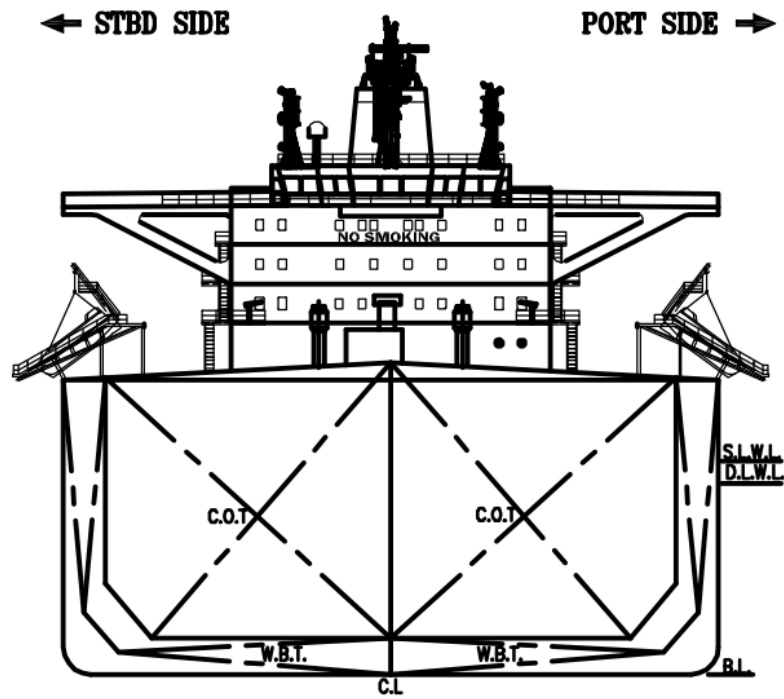


Figure 4-1 Case study vessel midship section.

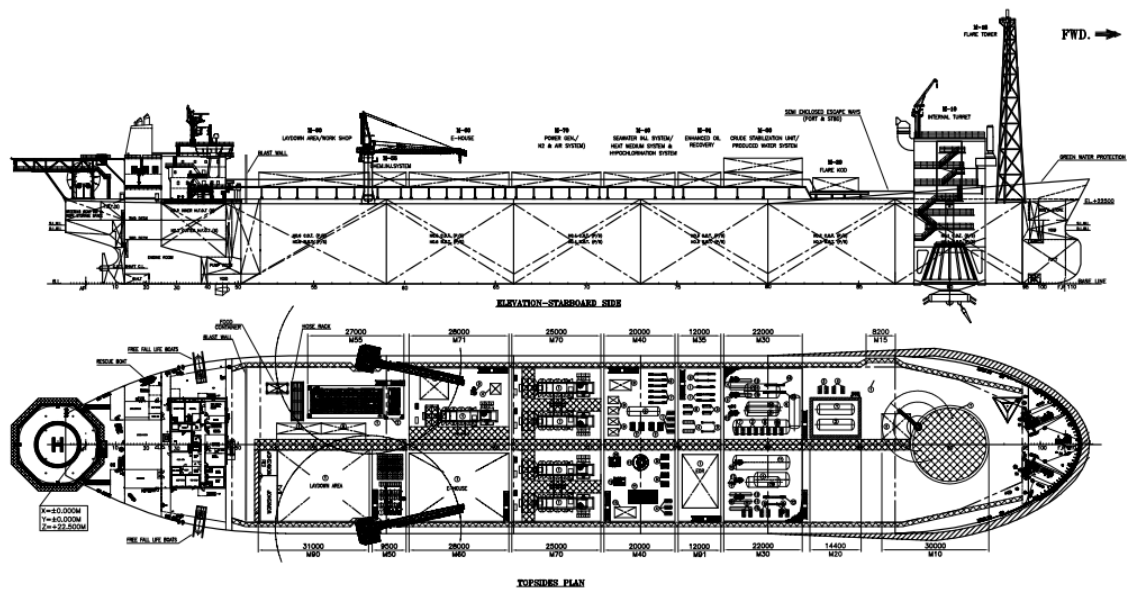


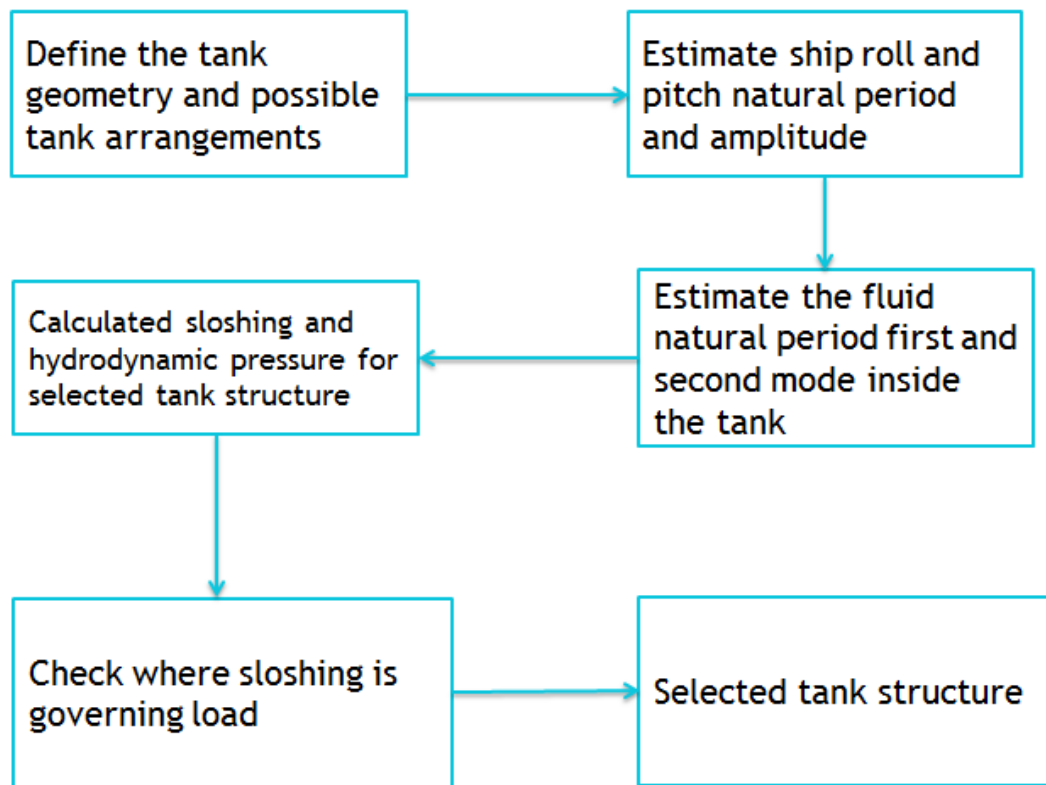
Figure 4-2 Case study vessel general arrangement.

## 4.2 Description of the Procedure

The case study in this thesis follows the procedure presented in figure 4-3. The procedure is based on an enumeration method and theory which is described in chapter 3. This procedure includes six steps. Those steps are:

1. Define the tank geometry and possible tanks structural arrangement.
2. Calculate the roll and pitch mode natural period and amplitude.
3. Calculate the fluid natural period in the tank.
4. Calculate sloshing and hydrodynamic pressure.
5. Check where sloshing is a governing load.
6. Make a design selection.

The first step is to define the tank geometry and possible tanks structural arrangements. Those possible arrangements provide the limits for the enumeration. Tank geometry also defines possible tank dimensions. One limitation is that the tank breadth is not greater than the tank length. This limitation gives the minimum length of the tank. This is a normal assumption because ship cargo tank length is larger than the tank breadth. Tank maximum length comes from ABS rules. The maximum tank length for a sloshing calculation is 54 meters. The rules allow longer tanks but in the case study vessel that means only one or two tanks in the longitudinal direction. The second step is to calculate the roll and pitch mode natural period and amplitude. The third step is to calculate the fluid natural period in the tank. The fourth step is to calculate sloshing pressure, using ABS rules. The fifth step is check where sloshing is a governing load. Usually maximum sloshing pressure is greater than hydrodynamic tank pressure at a low filling level. Finally a selection is made from all the possible options.



**Figure 4-3 Description of the Procedure.**

### **4.3 The tank geometry and arrangement**

Possible tank arrangements for enumeration are 3x2, 4x2, 5x2, 6x2, 7x2 and 8x2. When the maximum tank length is considered the number of tanks in the longitudinal direction is 3. Minimum tank length is tank breadth when tank length is 22 meters. In that case the number of tanks in the longitudinal direction is 8. The number of tanks in the transverse direction is constant because adding or removing tanks isn't practical and it is not cost efficient. Tank geometry is simple to define because the general tank shape is constant so tank geometry is same in all options.

## 4.4 Results

### 4.4.1 The natural period of fluid motion

The natural period of the fluid motion results in different tank size which are presented in figure 4-4 and 4-5. The results shows that Faltinsen's model and ABS rules give the same results for all tank size when the damping effect is missing. When adding a swash bulkhead to the tank, the natural period decreases a little; see figure 4-5. The ABS rules give a smaller result for natural period but Faltinsen's model doesn't take into account the effect of the swash bulkhead. The 3x2 tank size natural periods of fluid motion in the longitudinal direction are presented in figure 4-5. The natural period of the fluid motion decreases when the filling level rises. Faltinsen's model gives higher values than the ABS rules because ABS rules are taking into account the effect of the swash bulkheads. Now, the Faltinsen's model gives 23 percent higher values for all the tank sizes and filling levels.

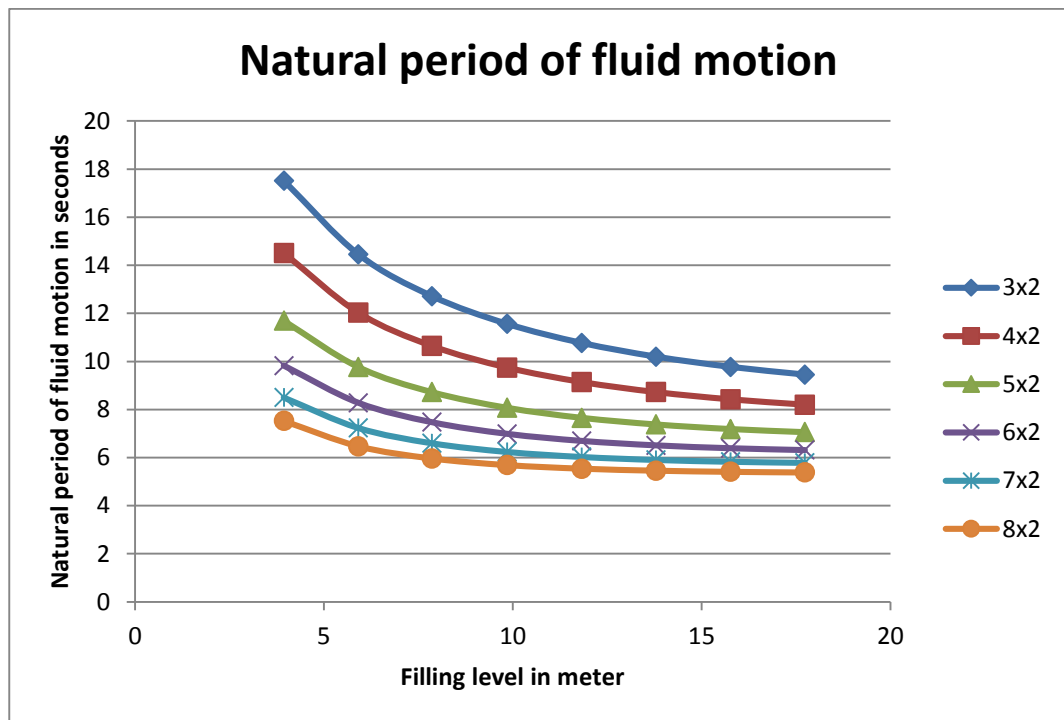
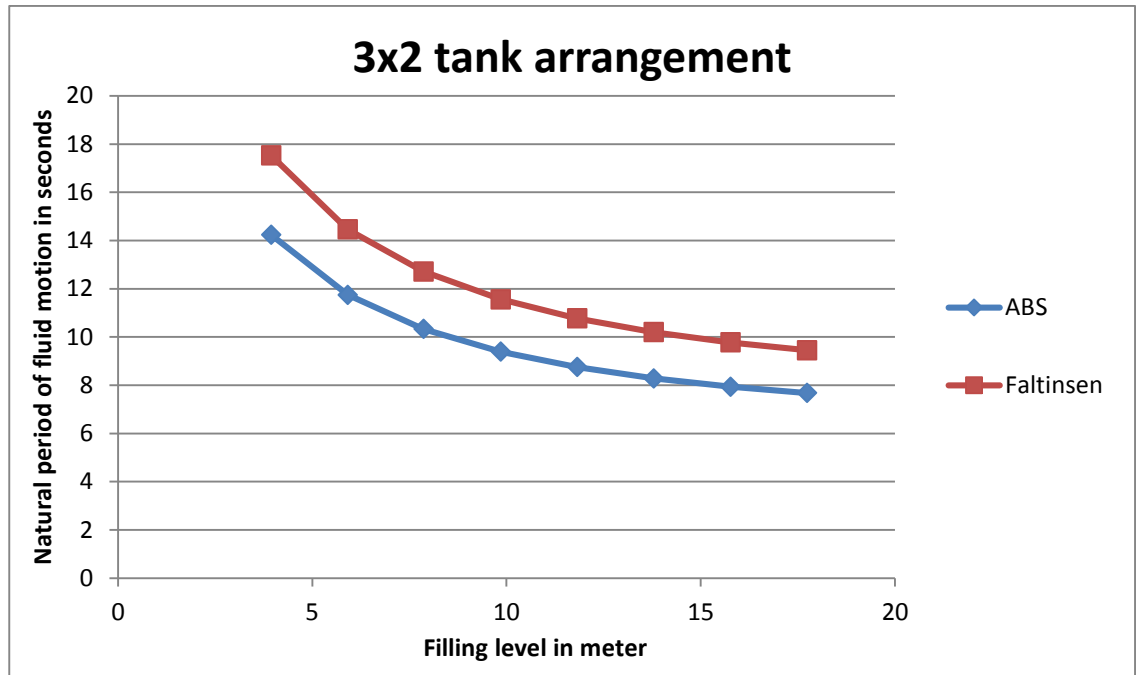


Figure 4-4 Natural period of fluid motion for all tank sizes.



**Figure 4-5 Natural period of fluid motion in 3x2 tank arrangement.**

In the figure 4-5 the red curve shows the results of the Faltinsen's model for the natural period of fluid motion. The blue one shows the results given by the ABS rules with a swash bulkhead. The natural period results show that ABS estimates the fluid motion in the tank by using a 2D model.

Secondary resonance is only relevant in the 3x2 and 4x2 tank arrangements when  $n$  is two. In other cases the fluid wave energy is too small to generate a resonance effect. In figure 4-6 a 3x2 tank arrangement is presented in roll and pitch mode. When the filling level is about 40 percent, secondary resonance occurs in pitch mode. In this case sloshing is possible.

The same happens for the 4x2 tank arrangement when the filling level is about 20 percent. This is presented in figure 4-7. Results also show that in a small tank roll mode starts to affect the secondary resonance effect. In this case sloshing is possible. In other cases the fluid start to resonate with the roll mode but the fluid wave



energy is too small. Because of this, resonance phenomenon doesn't cause a problem. Other secondary resonance results are presented in appendix 2.

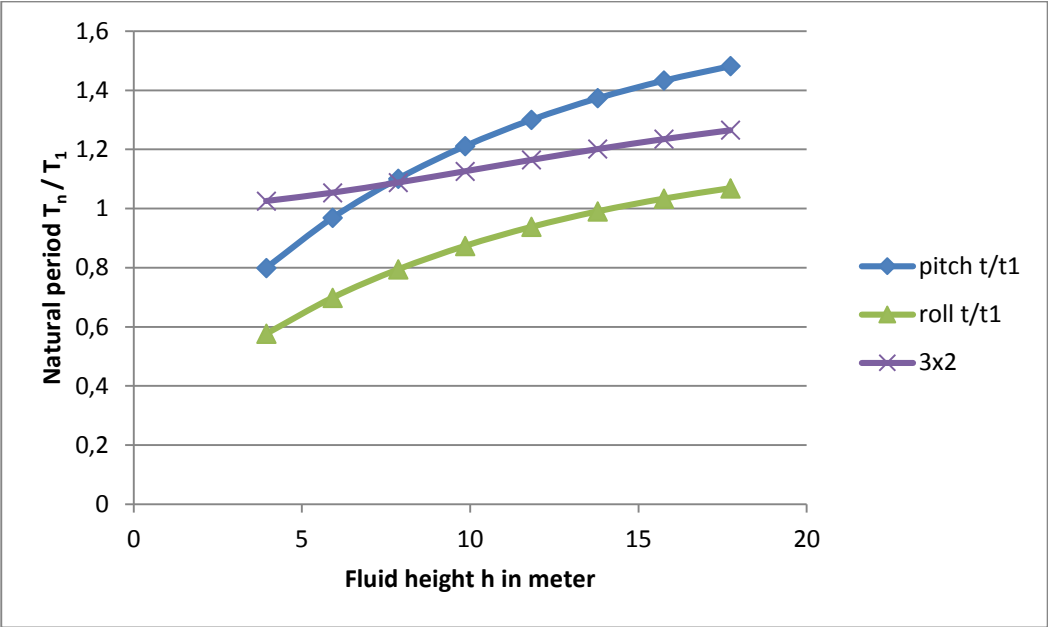


Figure 4-6 3x2 Tank arrangement secondary natural period in roll and pitch mode.

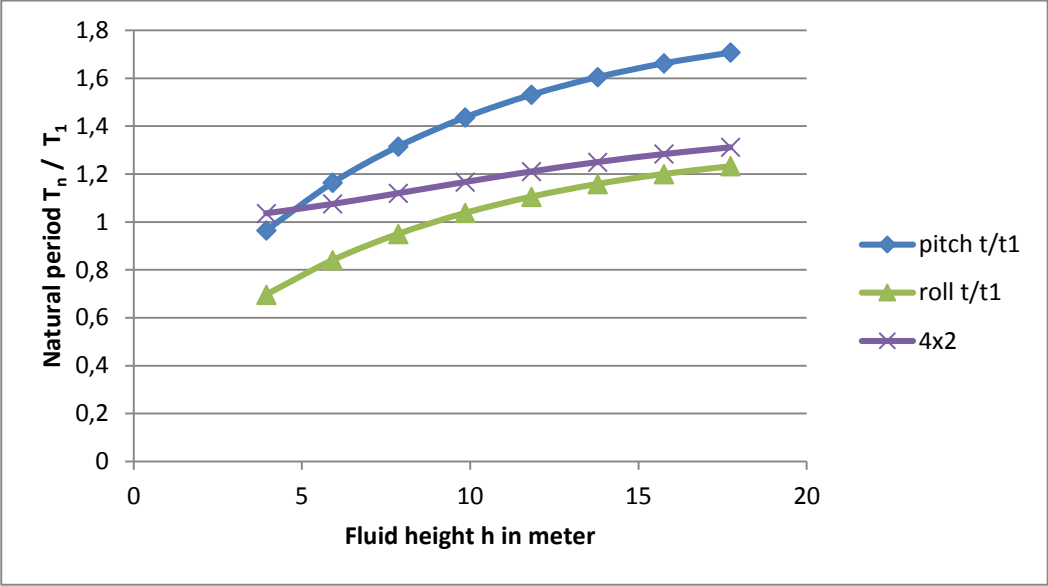


Figure 4-7 4x2 Tank arrangement secondary natural period in roll and pitch mode.

#### 4.4.2 Case vessel's motion analysis

Motion analysis gives acceleration in the tank, natural period and amplitude in roll and pitch mode. The calculation assumed three hour response. Sea area where the vessel operating is the North Atlantic. Results are presented in table 4-2 and table 4-3.

**Table 4-2 Case study ships roll and pitch mode natural period and amplitude.**

	Roll motion	Pitch motion
Natural period	10,1	14,0
Amplitude	9,0	6,4

**Table 4-3 Accelerations in cargo tanks.**

cargo tank	Surge(g)	Sway(g)	Heave(g)
T3	0.075	0.131	0.202
T4	0.074	0.132	0.15
T5	0.074	0.132	0.139

#### 4.4.3 Sloshing pressure

In figure 4-8 sloshing pressure is presented in the tank top structures when the cargo tank structure arrangements are 3x2, 4x2, 5x2 and 6x2. The results show that sloshing pressures can be reduced by optimizing the tank sizes but without making the tanks too small. Results also show that the effect of a swash bulkhead is significant when the tank length is large. In this case the effect of a swash bulk-head isn't significant when the tank length is 29,7 meters or less. In figure 4-8 the filling level represents the height of the liquid surface as a percentage. The normal curve indicates that the tank does not have a sloshing damping factor. The swash curve indicates that the full swash bulkhead is added to the tank. When the tank arrangement is 3x2, the normal curve sloshing peak pressure is when the filling level is at 30 percent. The swash curve peak pressure is when the filling level is at 20 percent. It is interesting to note that the peak pressure comes at various filling levels. This is expected because the effective sloshing length varies.

In the other cases sloshing peak pressure is achieved, when the filling level is 20 percent. In the tank structure 4x2 sloshing pressure is two times greater than without a swash bulkhead so the swash bulkheads effect is large. The effective length influence is grater now and the swash bulkhead is a more effective. When the tank is large, the swash bulkhead reduces the pressure significantly because the effective length relationship between swash and normal is huge. Sloshing pressure is low on the tank top structure, when the filling level is high.

When the cargo tank arrangement is 6x2, the swash bulkheads effect is much lower than in previous tank arrangements and sloshing pressure is not governing. Adding more tanks, has not achieved a sloshing pressure reduction. Other possible tank arrangement results are represented in appendix 3.

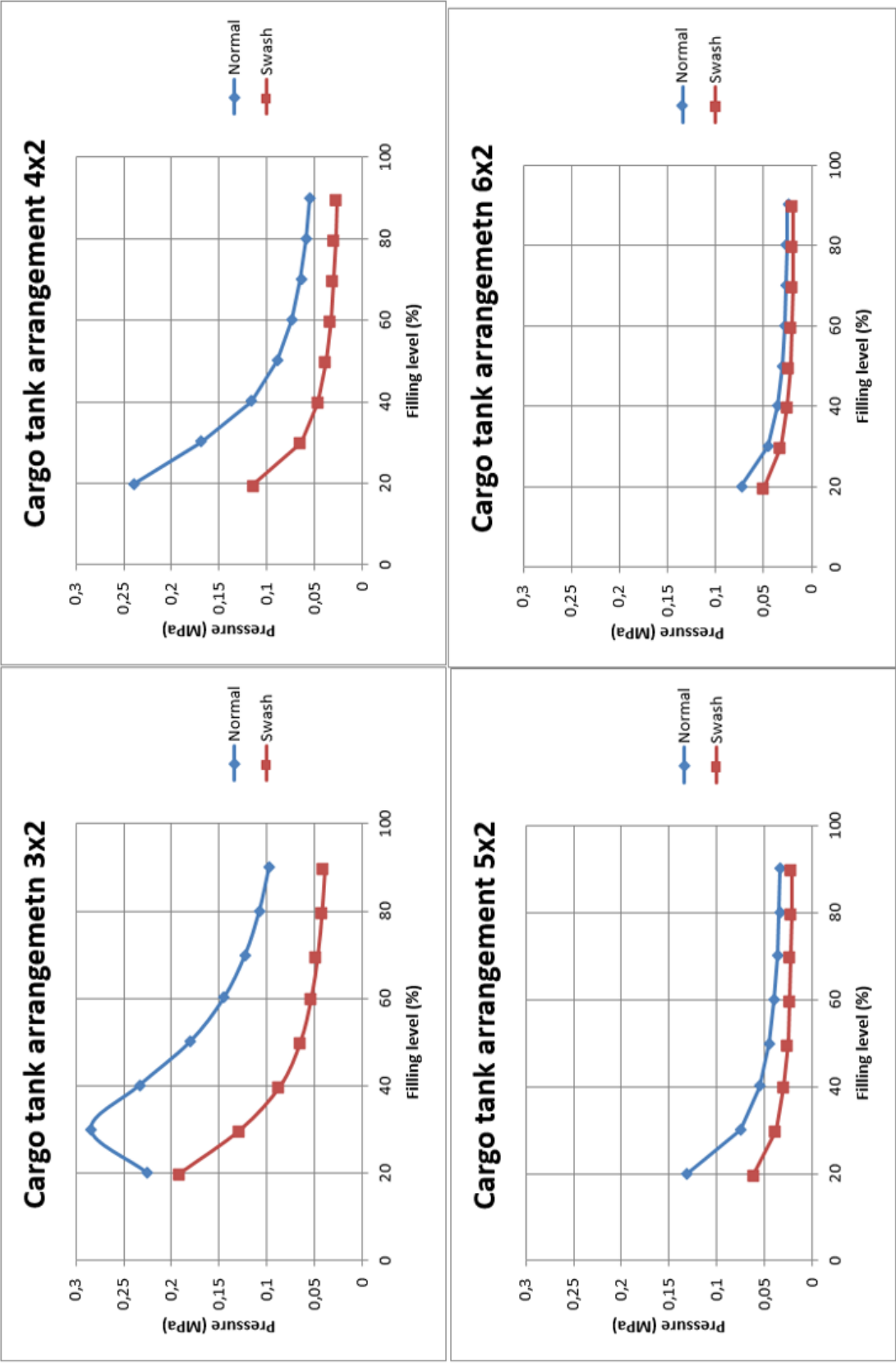
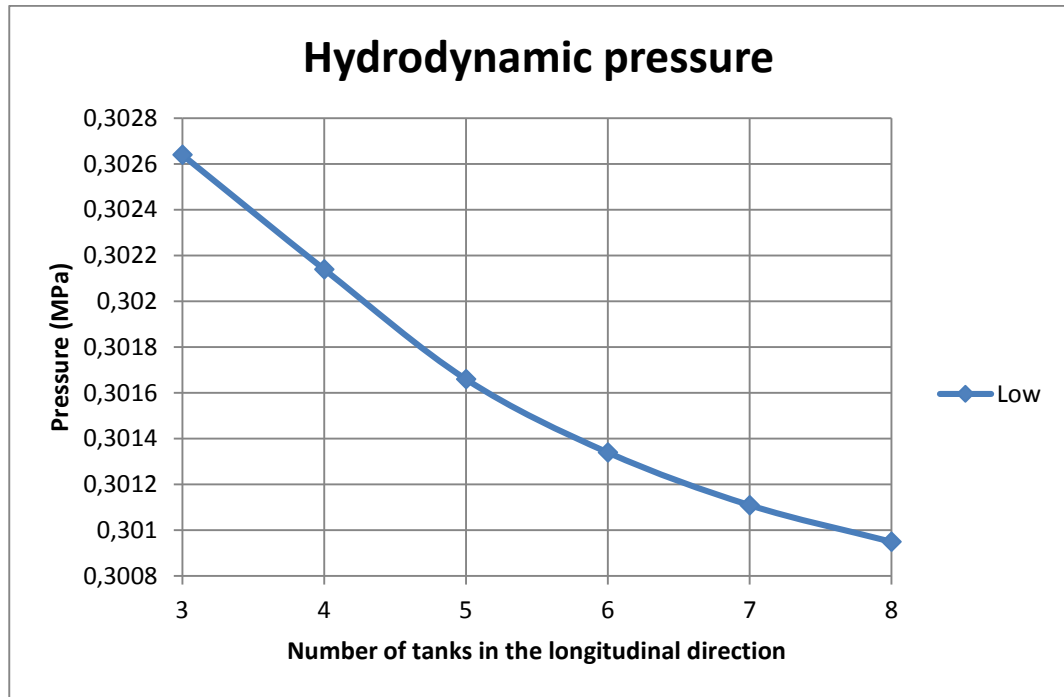


Figure 4-8 Sloshing pressure at various filling levels in the tank top structure.

#### 4.4.4 Tank structure hydrodynamic- and hydrostatic pressure

Tank structure hydrodynamic and hydrostatic pressure need to be calculated in order to know the cargo tank governing pressure. In figure 4-9 tank hydrodynamic pressure is presented. Hydrodynamic pressure decreases when the length of the tank decreases.

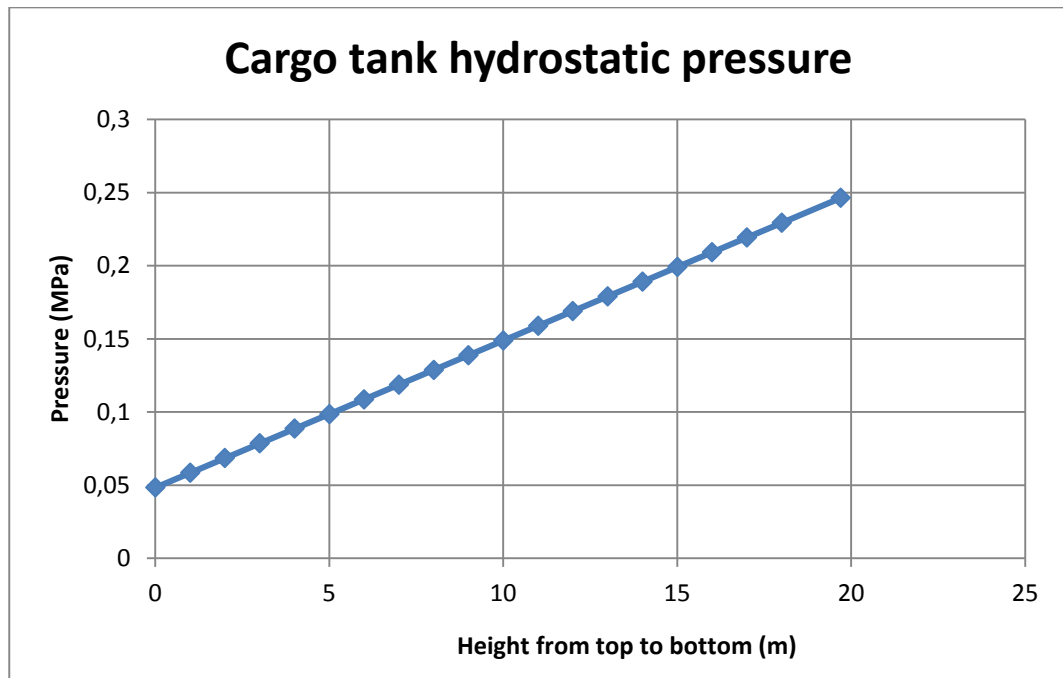


**Figure 4-9 Maximum hydrodynamic pressure.**

In the figure above the number of tanks describes the tank's length. Maximum hydrodynamic pressure affects the lower part of tank structure. Pressure varies between 0,3010 - 0,3026 MPa. Hydrodynamic pressure in the top of the tank structure varies between 0,2239 - 0,2244 MPa. Hydrodynamic pressure doesn't vary a lot because the accelerations are small.

In figure 4-10 tank hydrostatic pressure is presented. Hydrostatic pressure at the top of the tank structure is 0,09 – 0,10 MPa and at the lower part of the tank structure it is 0,25 MPa. The bulkhead plate thickness varies usually about three or four times

in the vertical direction. The top structure plate thickness is less than the lower structure.



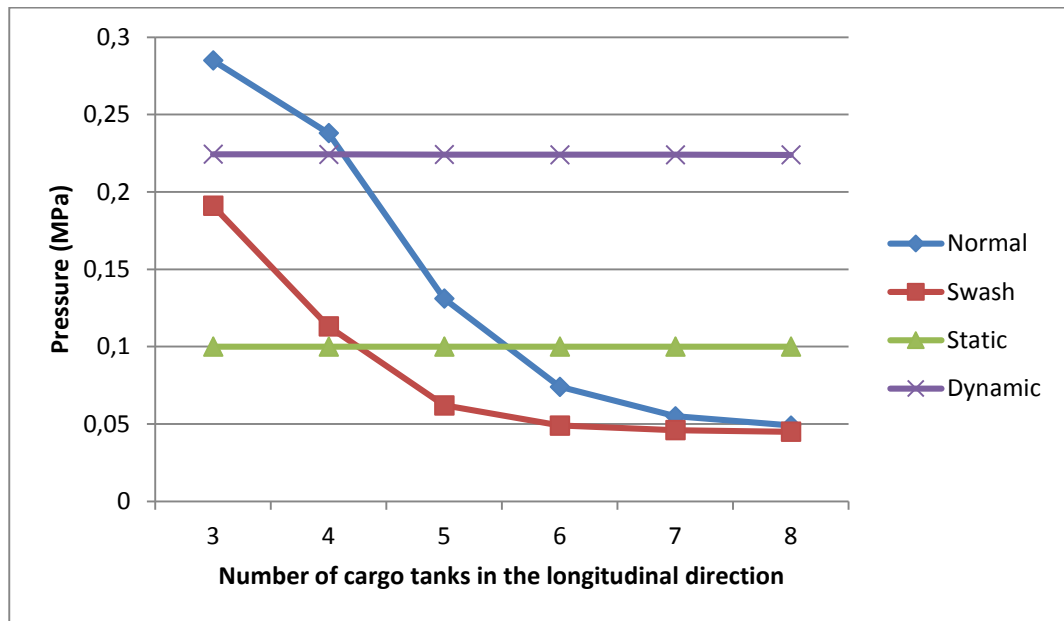
**Figure 4-10 Cargo tank hydrostatic pressure.**

Sloshing pressure is the top structures governing load when the pressure is greater than hydrodynamic pressure or hydrostatic pressure. At the top of the structure the limit is 0,2239 - 0,2244 MPa. The same limit for the lower structure is 0,3010 - 0,3026 MPa.

At the top of the tank structure sloshing pressure is the governing load for a tank arrangement 3x2, 4x2 without a swash bulkhead. For other cargo tank arrangements hydrodynamic pressure is the governing load. For the lower part of the tank structure sloshing pressure is the governing load for a tank arrangement 3x2 without a swash bulkhead.

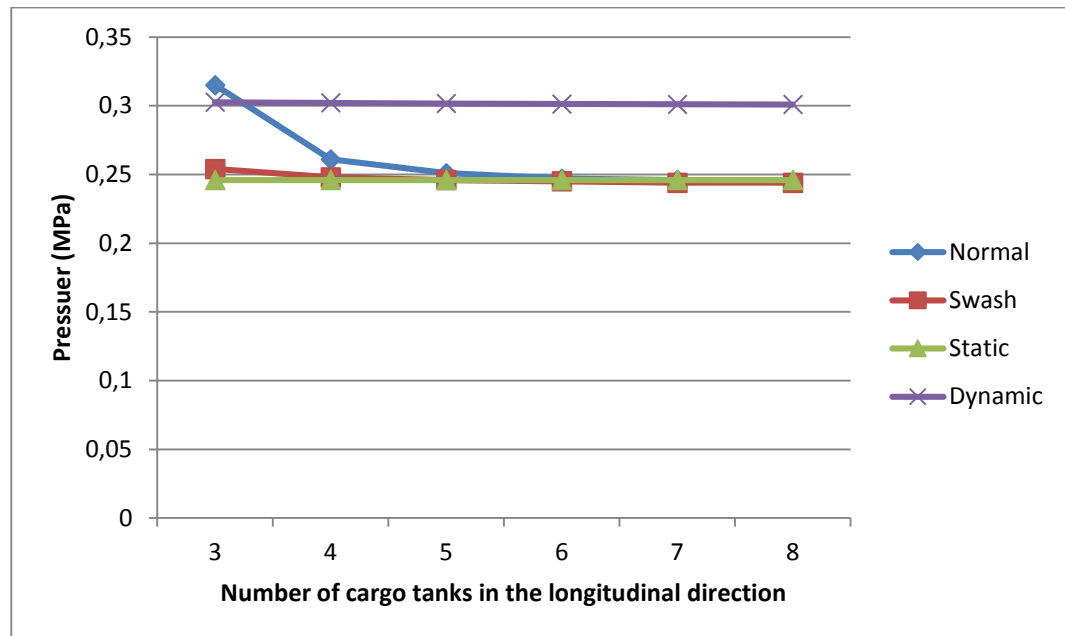
#### 4.4.5 Selected tank structure

Tank structure selection is made by using the calculated values. If we want to eliminate the sloshing pressure design criterion, we choose a tank arrangement where sloshing pressure is less than hydrodynamic pressure. If we want to control sloshing pressure, we choose a tank arrangements where sloshing is the governing load. Vessel's weight can affect the choice of the tank arrangement. Plate thickness affects the weight of the tank. Minimum plate thickness is 9.5 millimetre from ABS rules. A swash bulkhead adds more tank weight so it is not always the best solution to reduce the sloshing pressure. In figure 4-11 top of the cargo tanks pressure is presented and in figure 4-12 lower part of the cargo tanks pressure is shown. The normal curve presents sloshing pressure without a swash bulkhead, swash curve presents sloshing pressure with a swash bulkhead, dynamic curve presents hydrodynamic pressure and static curve presents hydrostatic pressure.



**Figure 4-11 Sloshing and internal pressure at the top of a cargo tank.**

Results show that sloshing pressure is governing load at the top of large tanks. Results also show that hydrodynamic pressure is the governing load at the top of smaller tanks.



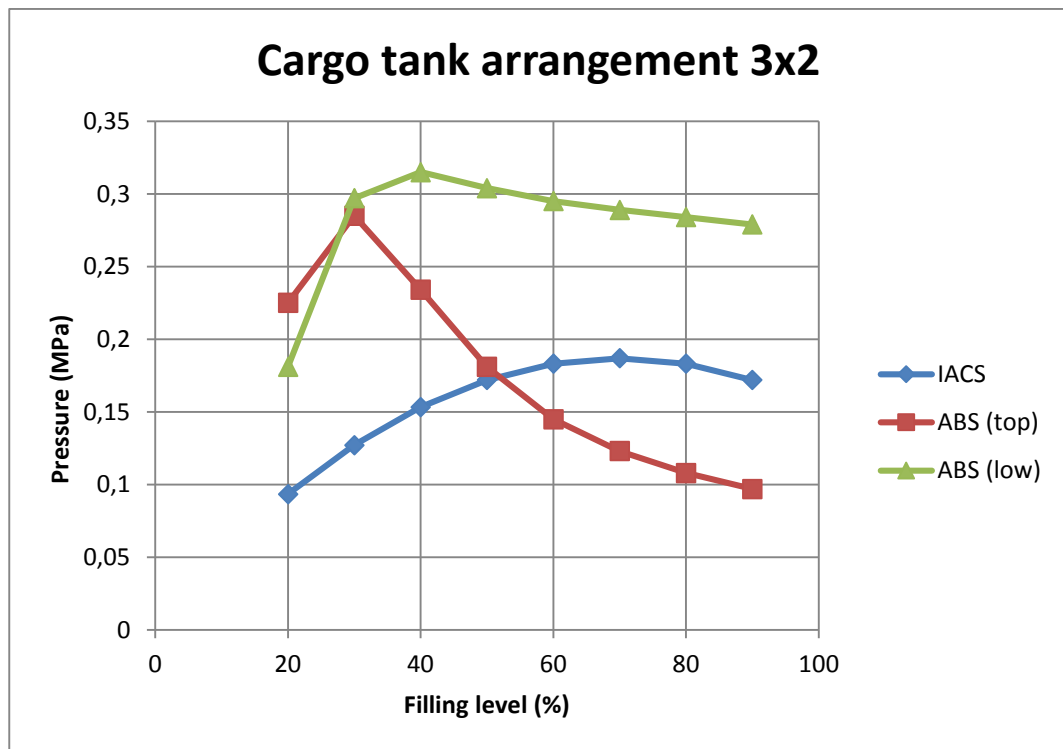
**Figure 4-12 Sloshing and internal pressure in the lower part of a cargo tank.**

In this case optimum solution for the top structure is a 5x2 tank arrangement without swash bulkheads. From a cost point of view the 5x2 tank arrangements is the best solution because the structures do not need to be changed. Changing the main structures is an expensive process because the main structure needs to be removed and built again. Sloshing pressure is greater than static pressure but less than hydrodynamic pressure. Plate thicknesses tell a lot about the weight and the bulkheads design. All options which are below the minimum plate thickness are not optimum because it is necessary to increase the plate thickness to 9,5 millimetre. Lower part plate thickness is almost the same. Plate thickness with swash bulkheads and without swash bulkheads is quite similar so the lower part does not affect the decision.



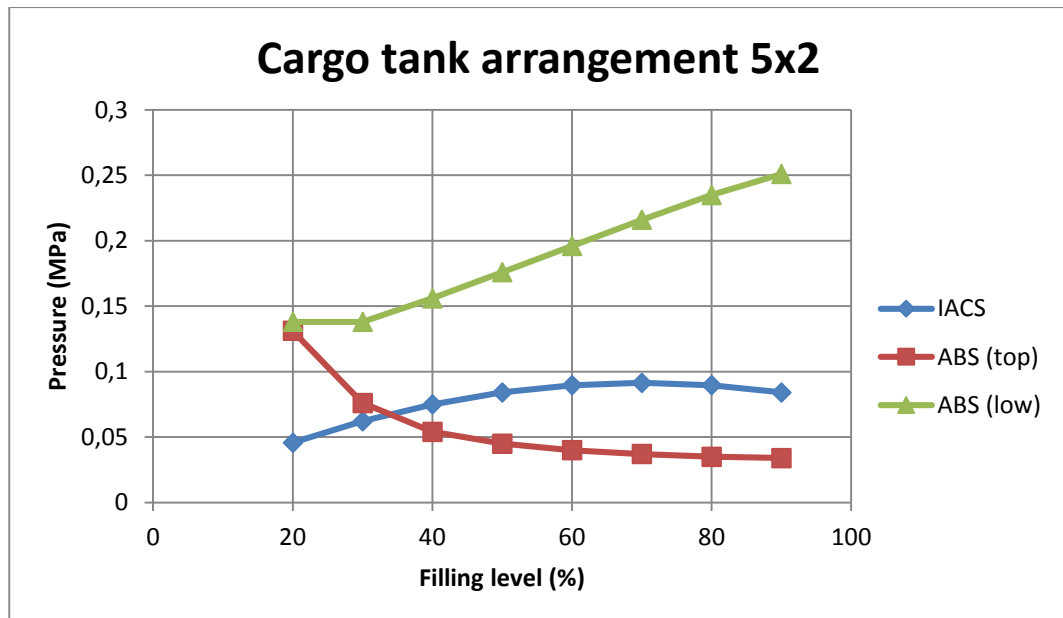
#### 4.4.6 Comparison of Classification Societies

Classification societies' sloshing pressure equations differ in many ways. The first intuition gives sloshing pressure almost the same regardless of classification societies. The case study vessel gives a good opportunity to compare sloshing pressure results. I chose two different cargo tank arrangements which are 3x2 and 5x2. The first tank arrangement is an interesting case because sloshing is governing loads. In figure 4-13 sloshing pressure in a cargo tank different filling levels is presented. In the figure there are three curves which are IACS, ABS top and ABS low. The IACS curve represents also the DNV and LR results because the sloshing equation is the same. ABS top represents the top structure sloshing pressure and ABS low represents sloshing pressure in the lower part of the structure.



**Figure 4-13 Different classification societies sloshing pressure results at various filling levels.**

The results are interesting because the intuition was proved wrong. ABS and IACS results differ in many ways. ABS results give the maximum sloshing pressure for the top structure when the filling level is at 30%. Then sloshing pressure starts to drop. IACS results give the maximum sloshing pressure when the filling level is at 70%. ABS low curve behaviour is interesting. If we compare ABS low and IACS curve shapes they are almost similar. When the filling level is small, sloshing pressure increases rapidly. When the filling level approaches 100%, the sloshing pressure part decreases and pressure is formed by the hydrostatic part. ABS top and IACS curves intersect when the filling level is at 50%. ABS and IACS results are inconsistent with each other. In figure 4-14 5x2 cargo tank arrangement's sloshing pressure is presented.



**Figure 4-14 Different classification societies sloshing pressure results at various filling levels.**

Figures 4-13 and 4-14 prove that the results are logical at different tank lengths. ABS top and IACS curve shapes are the same. ABS low curve is different but the reason is logical. In large tanks the sloshing pressure is greater than in small tanks. Figure 4-14 shows that ABS low curve begins to increase linearly when the filling

level is at 30%. This means that pressure's hydrostatic part is dominant in the lower part of the structure.

The first huge difference which affects the results is fluid motion. IACS doesn't take into account the exact fluid motion. ABS sloshing calculation allows for a more accurate value about fluid motion which can be solved e.g by using AQWA LINE program. The second difference is calculation points. ABS uses 27 point grids of which sloshing pressure can be calculated from. IACS uses only one point when sloshing pressure is calculated. For this reason, pressure calculations give different results for sloshing pressure. The second difference is how ABS and IACS define sloshing pressure. In both classification societies' sloshing pressure distribution is the same but the difference is how to calculate impact pressure. ABS calculates sloshing impact pressure over the full tank length in the upper part of the tank but IACS rules don't calculate.

## 5 Discussion

The purpose of this thesis was to develop a concept design phase calculation procedure for sloshing loads in a vessel's tank which provides information whether sloshing is the governing pressure in the tank. The calculation procedure solution is an enumerated method. The idea is to make limitations and calculate all possible tank size options. Based on the results the best option is then selected. The calculation procedure was developed by using ABS classification society's rules. ABS sloshing pressure equation terms have been improved by bringing acceleration and natural period values from the AQWA LINE program. This way the acceleration and natural period values are vessel-specific in defined sea conditions. Additionally, knowledge of the sloshing loads in a vessel's tank is increased.

When comparing different Classification Societies rules, the ABS rules are most comprehensive when calculating sloshing loads. All Classification societies have offshore rules but ABS is the only Classification Society that is meant for calculating FPSO vessel's sloshing loads. Other Classification Societies rules are meant for calculating tanker sloshing loads. That's why sloshing loads are not so exact because tanker loads are either full or empty. In these cases there is no variation in the liquid surface. The difference between those rules is the amount of calculation points and how the vessel's and fluid's natural periods are taken into account. ABS rules take both these into account.

When calculating a vessel's exact natural period, AQWA Line program can be used. When using that program there is no need to estimate vessel's natural period by using the rules of a Classification Society. Calculations are more specific. When calculating a fluid's natural period ABS equations can be used. Those equations take into account possible damping effects in the tank. If there is a double bottom and double sides in the tank there is no damping effect. When there is no damping effect in the tank, Faltinsen's 2D model is practical. In the case study vessel there is a double bottom and double sides in the tank so there are no damping effects. In

this case Faltinsen's model is justifiable to use. Results, given by ABS rules or Faltinsen's model, are the same. When adding a swash bulkhead to the tank structure, Faltinsen's model gave greater results for the fluid's natural period than ABS rules. This happened for all tank sizes and for all filling levels. In every case Faltinsen's model gave 23 percent greater values. The relative difference between natural periods remained equal the whole time. The reason why the results differ, is that assumptions differ between Faltinsen's model and Classification Society's rules.

Sloshing pressure results show that sloshing is a governing pressure in large tanks. In small tanks the governing load is the hydrodynamic pressure when the tank is completely filled. Sloshing pressure is a governing load in small tanks when the filling level is small. However, it is not the tank structures maximum design load. Results also show that different classification societies give different results for sloshing pressure. IACS, LR and DNV results are the same because the calculation procedures are the same but ABS gives higher values for sloshing pressure. Comparing sloshing pressure at the top of a tank structure of a 3x2 tank arrangement, ABS gives 52 percent higher values than DNV. DNV gives a maximum pressure value when the filling level is 70 percent. ABS gives a maximum pressure value when the filling level is at 20-30 percent. In this case the ABS results include only sloshing pressure because the calculation point is above the filling level.

From the case study vessel's results we can say when sloshing is a governing load in the vessel's tank and how the swash bulkhead affects the sloshing load. Results also show that sloshing pressure in the tank can be optimized and the right tank size can be found so that the tank is not too small. This thesis also indicates that sloshing isn't a major problem in the case study vessel's tanks because sloshing isn't a governing pressure in a 5x2 tank. In concept design work, classification society's rules are the only practical designing tools to calculate sloshing loads. Of course there are other methods which give more specific results and that's why they take much more time. However sloshing pressure can be optimized without those specific

methods in concept design work. Classification Societies rules with AQWA LINE results are exact enough when designing a vessel's tank structure.

Optimizing the tank arrangement for sloshing loads in concept design is unreal because other design parameters affect more in tank design. Usually FPSO is a conversion project where the tank structure area is constant. Normally this means that the vessel's tank area is not changed by adding more tanks. If sloshing is a problem in practice, add plates or swash bulkheads inside the tank.

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## Appendix 1 Sloshing pressure code

# Sloshing pressure

Load factor as defined in 2-1-3/5.7.2(a)

$$k_s := 0.878$$

Correlation factor for combined load cases and may be taken as unity unless otherwise specified

$$k_c := 1$$

Depth of tank, in m

$$h := 19.7$$

Distance, in m, measured from the tank bottom to the point considered

$$y := 19.7$$

Filling level, in m (ft), as shown in 2-1-3/Figure 9

Static head, taken as the vertical distance, in m (ft), measured from the filling level,  $d_m$ , down to the point considered.

$d_m$ , the filling level for maximum  $h_c$  calculated with  $C_{\phi s}$  and  $C_{\theta s}$  equal to 1.0, should not be taken less than  $0.55h$ .

$$d_m := 3.94$$

$$h_m := d_m = 3.94$$

Coefficient in accordance with 2-1-3/Figure 10

$$c_m := 1.25$$

$$c_{m2} := 1.5$$

$$c_{m3} := 1.25$$

weighted coefficients as given in 2-1-3/Figure 10

$$C_{\phi s} := 0$$

$$C_{\phi s2} := 1$$

$$C_{\phi s3} := 1$$

weighted coefficients as given in 2-1-3/Figure 10

$$C_{\theta s} := 1$$

$$C_{\theta s2} := 1$$

$$C_{\theta s3} := 0$$

Load factor, and may be taken as unity unless otherwise specified

$$k_u := 1$$

ESF for pitch amplitude, as defined in 2-1-1/1.1.1.

$$\beta_{\phi} := 1$$

ESF for roll amplitude, as defined in 2-1-1/1.1.1.

$$\beta_{\theta} := 1$$

represents  $\beta^*$  for transverse bulkheads

$$\beta_{tT} := 1$$

represents  $\beta^*$  for longitudinal bulkheads

$$\beta_{tL} := 1$$

$$b := 21.870$$

$$l_1 := 22.3$$

Density and gravitation constant is 1.005 N/cm<sup>3</sup>·m

$$\rho g := 1.005$$

effective tank length that accounts for the effect of deep ring-web frames, in m (ft)

$$l_e := \beta_{t_L}^2 \cdot l_1 = 22.3$$

effective tank width that accounts for the effect of deep ring-web frames, in m (ft)

$$b_e := \beta_{t_L}^2 \cdot b = 21.87$$

filling depth in m

$$d_o := d_m = 3.94$$

height of deep bottom transverses measured from the tank bottom, (2-1-3/Figure 13), in m

bottom height of the lowest openings in non-tight transverse bulkhead measured above the tank bottom

or top of bottom transverses (2-1-3/Figure 13), in m

$$d_{11} := 0$$

$$d_{12} := 0$$

number of deep bottom transverses in the tank

$$n := 0$$

height of deep bottom longitudinal girders measured from the tank bottom (2-1-3/Figure 13), in m

$$d_{b1} := 0$$

bottom height of the lowest openings in non-tight longitudinal bulkhead measured above the tank bottom,

or top of bottom longitudinal girders (2-1-3/Figure 13), in m

$$d_{b2} := 0$$

number of deep bottom longitudinal girders in the tank

$$m := 0$$

$$k_{11} := 1$$

$$k_{12} := 1$$

$$k_{b1} := 1$$

$$k_{b2} := 1$$

$$\sigma_n := \frac{\left(\frac{4}{\pi}\right) \cdot (n+1)}{\left[n \cdot (n+2) \cdot \cos\left[\frac{\pi}{2 \cdot (n+1)}\right]\right]}$$

$$\sigma_m := \frac{\left(\frac{4}{\pi}\right) \cdot (m+1)}{\left[m \cdot (m+2) \cdot \cos\left[\frac{\pi}{2 \cdot (m+1)}\right]\right]}$$

$$d_1 := d_o - d_{11} \cdot \left[1 - \sigma_n^2 \left(\frac{n+1}{2}\right)\right]^{0.5} \cdot k_{11} - 0.45 \cdot d_{12} \cdot k_{12}$$

$$d_1 := d_o = 3.94$$

$$d_b := d_o - d_{b1} \cdot \left[1 - \sigma_m^2 \left(\frac{m+1}{2}\right)\right]^{0.5} \cdot k_{b1} - 0.45 \cdot d_{b2} \cdot k_{b2}$$

$$d_b := d_o = 3.94$$

$$H_1 := h - d_{11} \cdot \left[ 1 - \sigma_n^2 \left( \frac{n+1}{2} \right) \right]^{0.5} \cdot k_{11} - 0.45 \cdot d_{12} \cdot k_{12}$$

$$H_1 := h = 19.7$$

$$H_b := h - d_{b1} \cdot \left[ 1 - \sigma_m^2 \left( \frac{m+1}{2} \right) \right]^{0.5} \cdot k_{b1} - 0.45 \cdot d_{b2} \cdot k_{12}$$

$$H_b := h = 19.7$$

The pitch natural period

$$T_p := 14$$

The roll natural period

$$T_r := 10.11$$

$$\beta_T := 0.66$$

$$\beta_L := 1$$

$$H_y := \pi \cdot \frac{d_b}{b_e} = 0.566$$

$$H_x := \pi \cdot \frac{d_l}{l_e} = 0.555$$

$$k_y := \left( \frac{\tanh(H_y)}{4 \cdot \frac{\pi}{9.81}} \right)^{0.5} = 0.632$$

$$k_x := \left( \frac{\tanh(H_x)}{4 \cdot \frac{\pi}{9.81}} \right)^{0.5} = 0.627$$

The natural period of the fluid motion in the longitudinal direction

$$T_x := \frac{(\beta_T \cdot l_e)^{0.5}}{k_x} = 6.114$$

The natural period of the fluid motion in the transverse direction

$$T_y := \frac{(\beta_L \cdot b_e)^{0.5}}{k_y} = 7.394$$

$$x_o := \frac{T_x}{T_p} = 0.437$$

$$x_{o1} := x_o = 0.437$$

$$y_o := \frac{T_y}{T_r} = 0.731$$

$$y_{o1} := y_o = 0.731$$

$$C_{fl} := 0.792 \cdot \left[ \frac{d_l}{(\beta_T \cdot 1_e)} \right]^{0.5} + 1.98 = 2.39$$

$$C_{fb} := 0.704 \cdot \left[ \frac{d_b}{(\beta_L \cdot b_e)} \right]^{0.5} + 1.76 = 2.059$$

$$C_{tl} := 0.9 \cdot \frac{x_{o1}}{\left[ 1 + 9 \cdot (1 - x_o)^2 \right]} = 0.102$$

$$0.9 \cdot \frac{x_{o1}}{\left[ 1 + 9 \cdot (1 - x_o)^2 \right]} \geq 0.25$$

$$C_{tb} := 0.9 \cdot \frac{y_{o1}}{\left[ 1 + 9 \cdot (1 - y_o)^2 \right]} = 0.399$$

$$0.9 \cdot \frac{y_{o1}}{\left[ 1 + 9 \cdot (1 - y_o)^2 \right]} \geq 0.25$$

$$\phi := 6.4$$

$$\theta := 9$$

$$\phi_{es} := 0.71 \cdot \phi = 4.544$$

$$\theta_{es} := 0.71 \cdot \theta = 6.39$$

$$h_l := \phi_{es} \cdot 1_e \cdot C_{tl} \cdot \beta_T \cdot \left[ 0.018 + C_{fl} \cdot \frac{\left( 1.0 - \frac{d_l}{H_l} \right)}{\phi_{es}} \right] \cdot \frac{d_l}{H_l} = 0.598$$

$$h_b := \theta_{es} \cdot b_e \cdot C_{tb} \cdot \beta_L \cdot \left[ 0.016 + C_{fb} \cdot \frac{\left( 1.0 - \frac{d_b}{H_b} \right)}{\theta_{es}} \right] \cdot \frac{d_b}{H_b} = 3.053$$

m (ft) for  $\theta_{es} > 6$  degrees

$$h_p := 1_l \cdot \sin(\phi_{es}) = -21.985$$

$$h_{tl} := 0.0068 \cdot \beta_T \cdot 1_e \cdot C_{tl} \cdot (\phi_{es} + 40) \cdot (\phi_{es})^{0.5} = 0.969$$

$$h_r := b \cdot \sin(\theta_{es}) = 2.332$$

$$h_{tb} := 0.0055 \cdot \beta_L \cdot b_e \cdot C_{tb} \cdot (\theta_{es} + 35) \cdot (\theta_{es})^{0.5} = 5.022$$

$$h_{tr} := k_c \cdot \left( C_{\phi s} \cdot h_{tl}^2 + C_{\theta s} \cdot h_r^2 \right)^{0.5} = 2.332$$

maximum average sloshing pressure heads, in m (ft), to be obtained from calculations as specified below for at least two filling levels, 0.55h and

the one closest to the resonant period of ship's motions, between 0.2h and 0.9h.h<sub>c</sub> may be taken as constant over the tank depth, h (See 2-1-3/Figure 9)

$$h_c := k_c \cdot \left( C_{\phi s} \cdot h_l^2 + C_{\theta s} \cdot h_b^2 \right)^{0.5} = 3.053$$

$$h_{c2} := k_c \cdot (C_{\phi s2} \cdot h_1^2 + C_{\theta s2} \cdot h_b^2)^{0.5} = 3.111$$

$$h_{c3} := k_c \cdot (C_{\phi s3} \cdot h_1^2 + C_{\theta s3} \cdot h_b^2)^{0.5} = 0.598$$

sloshing pressure heads for upper bulkhead, in m

$$h_{t2} := k_c \cdot (C_{\phi s2} \cdot h_{tl}^2 + C_{\theta s2} \cdot h_{tb}^2)^{0.5} = 5.115$$

$$h_t := k_c \cdot (C_{\phi s} \cdot h_{tl}^2 + C_{\theta s} \cdot h_{tb}^2)^{0.5} = 5.022$$

$$h_{t3} := k_c \cdot (C_{\phi s3} \cdot h_{tl}^2 + C_{\theta s3} \cdot h_{tb}^2)^{0.5} = 0.969$$

$$h_{e1} := k_u \cdot \left[ h_c + (h_t - h_c) \cdot \frac{(y - d_m)}{(h - d_m)} \right] = 5.022$$

$$h_{e2} := k_u \cdot \left[ h_{c2} + (h_{t2} - h_{c2}) \cdot \frac{(y - d_m)}{(h - d_m)} \right] = 5.115$$

$$h_{e3} := k_u \cdot \left[ h_{c3} + (h_{t3} - h_{c3}) \cdot \frac{(y - d_m)}{(h - d_m)} \right] = 0.969$$

for  $y > d_m$

$$h_{e22} := c_{m2} \cdot h_m + k_u \cdot h_{c2} = 9.021$$

$$h_{e32} := c_{m3} \cdot h_m + k_u \cdot h_{c3} = 5.523$$

for  $0.15h \leq y \leq d_m$  ( $c_m h_m$  need not exceed  $h$ )

$$h_{e12} := c_m \cdot h_m + k_u \cdot h_c = 7.978$$

$$h_{e23} := k_u \cdot h_{c2} = 3.111$$

$$h_{e33} := k_u \cdot h_{c3} = 0.598$$

$h_e$  calculated at  $y = 0.15h$  for  $y < 0.15h$ , but  $h_e$  should not be smaller than  $c_m h_m$

$$h_{e13} := k_u \cdot h_c = 3.053$$

$$h_u := c_m \cdot h_m = 4.925$$

$$p_{is} := k_s \cdot \rho g \cdot h_e = \blacksquare$$

for  $y > d_m$

$$p_{is2} := k_s \cdot \rho g \cdot h_{e2} = 4.513$$

for  $0.15h \leq y \leq d_m$

$$p1 := k_s \cdot \rho g \cdot h_{e12} = 7.04$$

$$p4 := k_s \cdot \rho g \cdot h_{e22} = 7.96$$

$$p7 := k_s \cdot \rho g \cdot h_{e32} = 4.874$$

$$p8 := k_s \cdot \rho g \cdot h_{e33} = 0.528$$

$$p2 := k_s \cdot \rho g \cdot h_{e13} = 2.694$$

$$p5 := k_s \cdot \rho g \cdot h_{e23} = 2.746$$

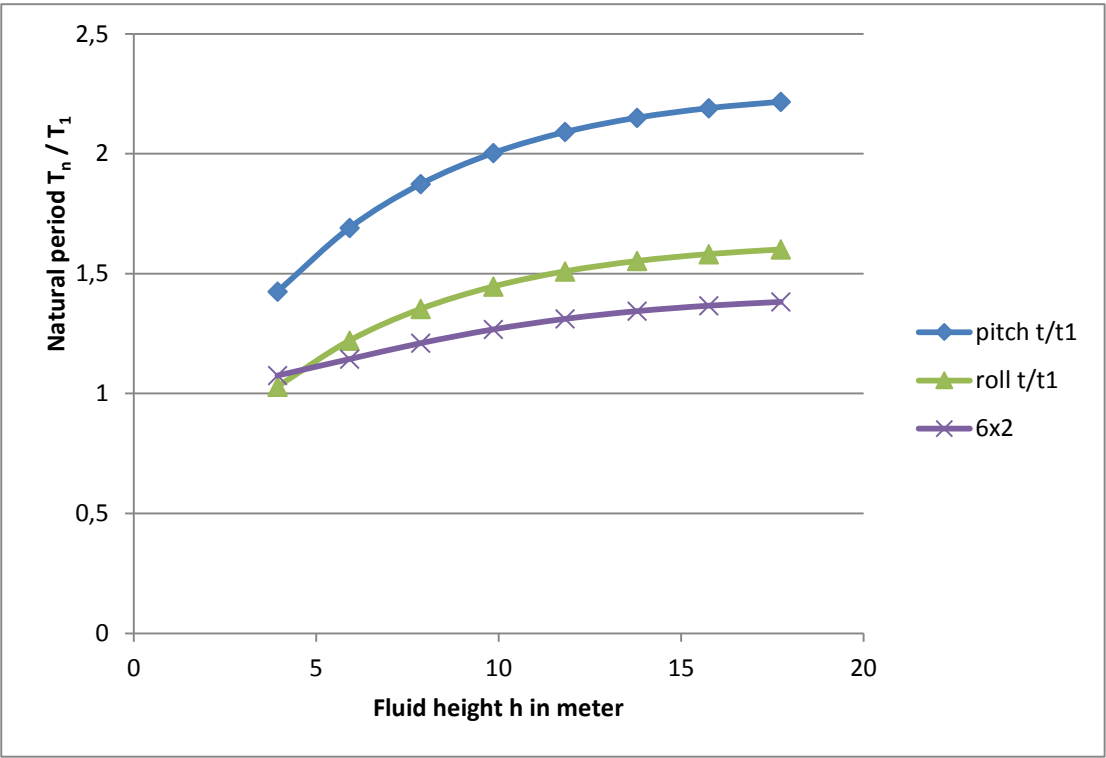
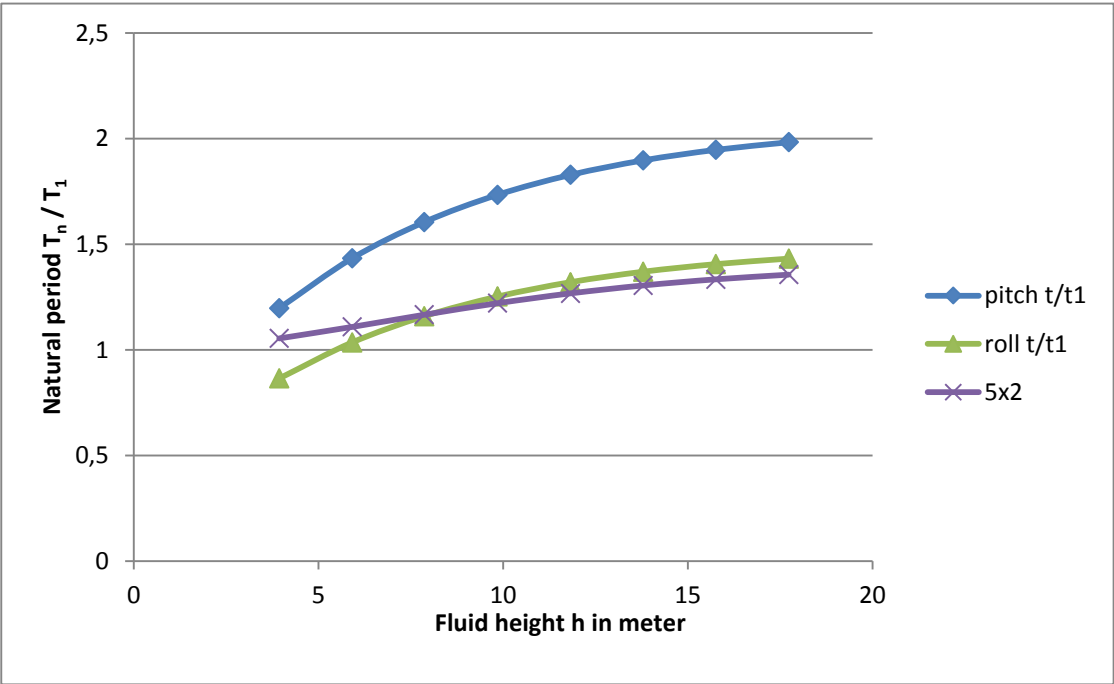
$$p3 := k_s \cdot \rho g \cdot h_{e1} = 4.432$$

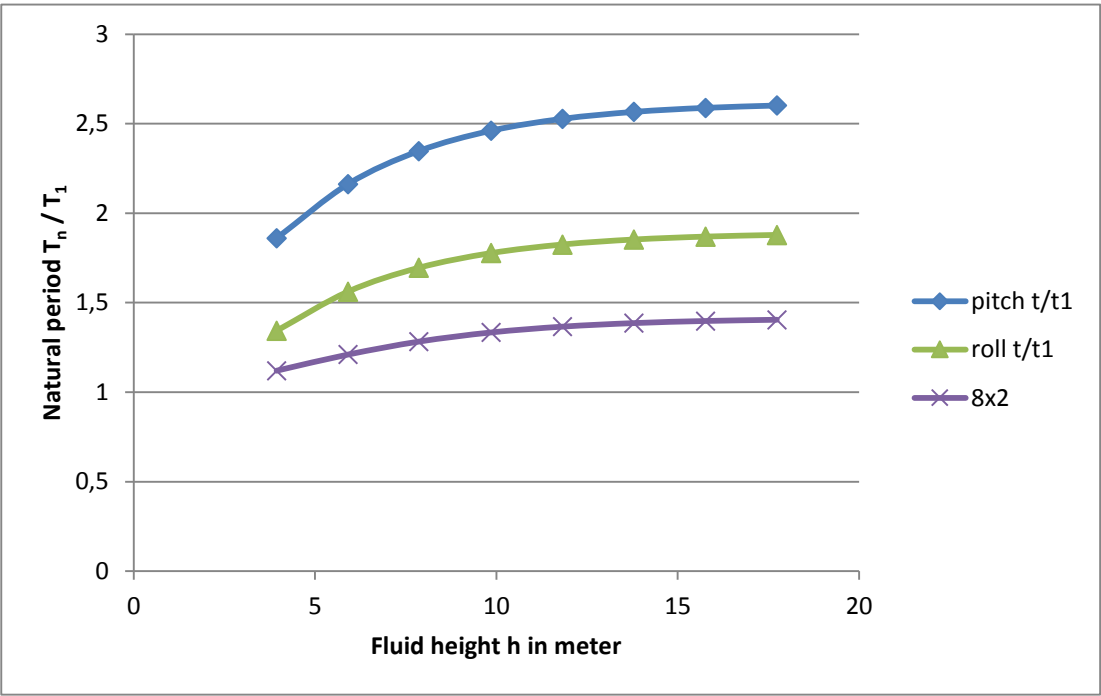
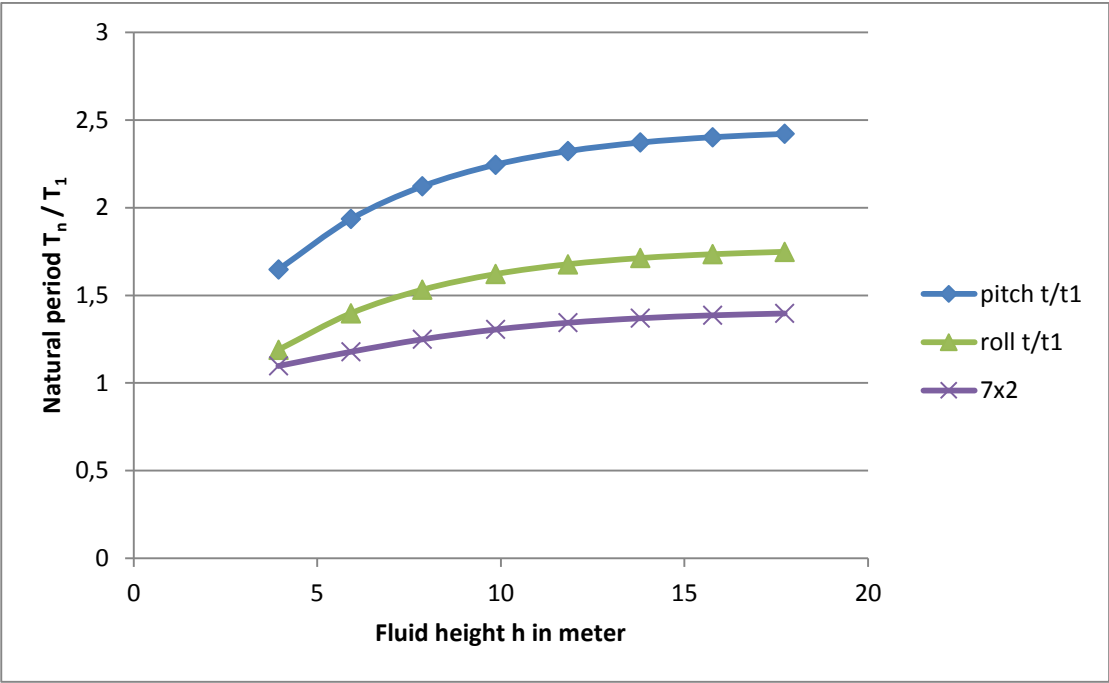
$$p6 := k_s \cdot \rho g \cdot h_{e2} = 4.513$$

$$p9 := k_s \cdot \rho g \cdot h_{e3} = 0.855$$



Appendix 2 Secondary resonance results





Appendix 3 Sloshing pressure results

